PROBLEM SOLVING PATHWAY

Problem Solving, The Fun Way

Selected Problems

Ashley C. Fernandes

Problem Solving Pathway Selected Problems

Problem Solving, The Fun Way

Selected Problems

Ashley C. Fernandes

Problem Solving Pathway: Selected Problems

ISBN-13 Applied for

Typesetting and drawing of all figures in LaTeX, by the author.

Disclaimer
Puzzles, games and quotations used in the books were checked for copyright permissions, by a professional Permissions Editor. Most are 'fair use', or sufficiently transformative and not needing permission. Wherever applicable, all reasonable effort was made to track original copyright holders for seeking permission.

While every effort was made to ensure error-free content and accurate facts, the complexity of the books makes minor glitches inevitable. A Technical Editor has reviewed the books, but errors or omissions in the content remain the author's responsibility.

Please get in touch, to improve the books. Anything reproduced without proper attribution or acknowledgment, or any legitimate error and/or omissions will be corrected at the first opportunity.

Companion Website
`www.problemsolvingpathway.com`

Contact
`info@problemsolvingpathway.com`

Welcome to the **Problem Solving Pathway**!

These selected problems are extracted from the first two books of the 'Problem Solving Pathway' series. The book series explores the 'fun' aspect of problem solving.

The first four books teach aspects of problem solving, using fun activities like puzzles and games. Here one finds constrained ways of solving a problem, therefore defined kinds of solutions that are acceptable.
The last three books apply insight gained by solving puzzles and playing games, to real world problems. Here, loosely constrained problems may have many ways to solve them. Acceptance of a solution may not only depend on how optimal or practical it is, but also on what real world humans will accept and implement.

A synopsis of all the books in the series is available at the end of this book.
Further details are available on the companion website : '`www.problemsolvingpathway.com`'.

Goals of the book series:
Make learning problem solving,

- A fun activity and not a chore
- Easy to understand, for anyone with high school education
- Memorable, to know how to use the tools in real life
- Cost-effective printed books (under $5 per printed book)

Intended Audience:
Mature individuals prepared to invest time and effort in learning:

- Anyone with a basic high school level of education
- College students
- STEM's (Science, Technical, Engineering, Mathematics)
- NEET's (Not in Education, Employment, Training)
- Employed individuals

The sample problems and their solutions may contain links or references to other problems from the book. The formatting in this ebook is different from that in the actual books. This is to be expected. Please ignore discrepancies in this sample compilation.

Think of this booklet as a mosaic – you need all the pieces in place to appreciate the entire picture. Solve these problems linearly from beginning to end. Later problems rely on the preceding ones, to build up the mosaic. Using this step-by-step approach, you will grasp hidden patterns and gain deep insights into problems, solutions, and decision making.

Doing, Not Reading

This is not a reading book, this is a doing book. To benefit, you must think *independently*, try to solve each problem *on your own*. Merely reading through the book will not make you a problem solver. If you get stuck, by all means cheat. Glance through the analysis and solutions outlined. And even if you get a correct solution on your own, do not neglect the detailed explanation.

This is a self-learning book, each explanation is detailed to the point of being long-winded! And that is the way this book series is designed. In the absence of a human instructor, the book will provide all the explanation to all levels of readers.

Each problem is dissected and analyzed in detail, to give you a rich understanding of the solution from the problem solving perspective. Invest time in understanding what kind of solution is expected and acceptable. Focus on proper analysis, and 90+% of the problem gets solved. Proper analysis prevents rework and random attempts (that may not lead to solutions). Pause and thoroughly understand each problem, its analysis, its solutions, all the learning the problem provides. Only then will you fully benefit from tackling the next problem along the pathway.

Look for multiple solutions to problems. Pick the one that is optimal (the best solution, given the current circumstances).

On the path to becoming a problem solver that can tackle *any* type of problem (and solve most of them satisfactorily), your thinking needs a lot of help and nudging to imbibe the problem solving mindset of analysis, investigation, logic, and judgment.

Thankfully, these books assist you on your way to mastering the art and science of problem solving.

While solving puzzles, do not lose focus of your goal – learning aspects of problem solving. Puzzles are tools to achieve the goal.

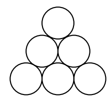
Solve *all* problems on a given page before turning the page over to see the explanations and detailed analysis.

This first set of mathematics problems are easy (they truly are). The goal is not to master mathematics, but to develop your logical and analytical faculties. The discipline and systematic thinking gained by learning mathematics will help you solve any (mathematical *or* non-mathematical) problem.

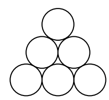
Triangulating Numbers

Only use positive whole numbers, one number per circle.

A: Put 1, 2, or 3, per circle so that the sum of each triangle side totals 5. Find five distinct solutions!



B: Put a different number from 1 to 6, no repeats, in each of six circles. The sum along each triangle side must total 9. Find all possible solutions.



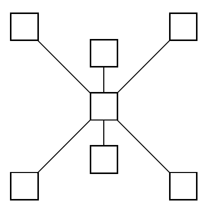
Frame The Numbers

Only use positive whole numbers, one number per cell.

C: Fill blank cells with numbers 1–9 (each appearing only once). Numbers must total the same in the horizontal (each row), vertical (each column), and in both major diagonal directions.

	3	
1		
		2

D: Triskadekaphilia Put numbers between 1–9, no repeats, in each of the seven cells. The total in the single (vertical) column and both (NE, NW) diagonals, must total 13.



A Counting Spree

E: At a traffic signal there were trucks and cars. Each car had four tires, each truck had 6. The total number of tires was 36. How many cars and trucks were at the signal?

F: What is the solution to Problem **E** if you count the spare tires, each vehicle has one?

Detailed analysis starts on page 8

Triangulating Numbers

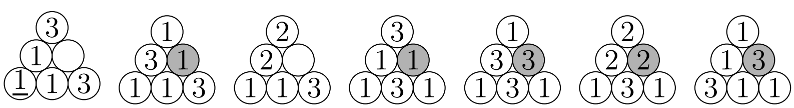
A: With six circles to fill with 1, 2, or 3, numbers *must* repeat. List all possible three number combinations of 1, 2, 3. Eliminate duplicate combinations and those that do not total exactly 5.

1 + 1 + (1, 2, 3)	1 + 2 + (1, 2, 3)	1 + 3 + (1, 2, 3)
2 + 1 + (1, 2, 3)	2 + 2 + (1, 2, 3)	2 + 3 + (1, 2, 3)
3 + 1 + (1, 2, 3)	3 + 2 + (1, 2, 3)	3 + 3 + (1, 2, 3)

A better way to find three number combinations totaling 5
Take the first number of three:
a 3: need two more in two numbers; must be (1, 1)
b 2: need three more in two numbers; must be (2, 1)
c 1: need four more; (3, 1), same as **a**! or (2, 2) same as **b**!

Only two valid combinations give 5: 1, 1, 3 and 2, 2, 1.
Put each combination on a straight line in different ways:
d: 1, 1, 3; **e**: 1, 3, 1; **f**: 3, 1, 1; **g**: 2, 2, 1; **h**: 2, 1, 2; **i**: 1, 2, 2.
d, **f** (and **g**, **i**) are mirrored combinations of each other.

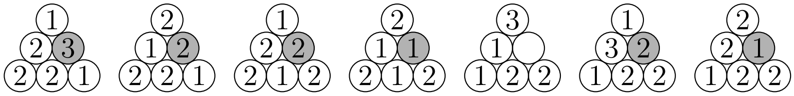
Map the three sides to *directions* NE, NW, E.
First put numbers on the E side of the triangle that fulfills the criteria of three circles summing to 5. Then put numbers on the NE side. The last number (in the circle shaded gray) in the middle of the NW side defines if a solution is valid (fulfills criteria of NW summing to 5). Try each option **d–i**, in turn.
Note: **a–c** valid combinations have **d–i** straight line combinations; E, NE, NW are triangle sides; **j–w** are figures.



j ✗ **k** ✓ **l** ✗ **m** ✓ **n** ✓ **o** ✓ **p** ✓

For E = **d**, 1 (underlined) is the bottom-left circle of the triangle.
From **c**: (3, 1) & (2, 2) fulfill criteria for NE side (figures **j**, **k**, **l**).
Figures **m**, **n**, **o** show results for E = **e**, referencing **c** (NE side).
Figure **p** shows results for E = **f**, referencing **a** (NE side).
j: E = **d**, NE = **d**: NW already totals 6, no solution
k: E = **d**, NE = **e**, **Solution** (NW = **f**)
l: E = **d**, NE = **i**: NW already totals 5, no solution
m: E = **e**, NE = **d**, **Solution** (NW = **d**)
n: E = **e**, NE = **e**, **Solution** (NW = **e**)
o: E = **e**, NE = **i**, **Solution** (NW = **i**)
p: E = **f**, NE = **f**, **Solution** (NW = **e**)

q–w illustrate options **g**, **h**, **i**, giving valid solutions (except **u**).



q ✓ **r** ✓ **s** ✓ **t** ✓ **u** ✗ **v** ✓ **w** ✓

Solutions **k** (= **m** = **p**), **n**, **o** (= **q** = **v**), **r** (= **s** = **w**), and **t**, are unique. Rotate the triangle to get equivalent solutions.

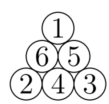
2 at an apex? 3 at second or third or both vertices, is illegal.
3 at an apex? 2 or 3 at second or third or both vertices, is illegal.
n and **t**: All three vertices and middle circles have same numbers. This makes all three sides symmetrical.

B: From the number pool (1–6), find three groups of three numbers each (that total 9). Consider each number **1–6**, in turn:

Number	Pool	Combination 1		Combination 2
1 + 8:	2, 3, 4, 5, 6	**1** + 2 + 6	**a**	**1** + 3 + 5 **b**
2 + 7:	1, 3, 4, 5, 6	**2** + 1 + 6	**a**	**2** + 3 + 4 **c**
3 + 6:	1, 2, 4, 5, 6	**3** + 1 + 5	**b**	**3** + 2 + 4 **c**
4 + 5:	1, 2, 3, 5, 6	**4** + 2 + 3	**c**	
5 + 4:	1, 2, 3, 4, 6	**5** + 1 + 3	**b**	
6 + 3:	1, 2, 3, 4, 5	**6** + 1 + 2	**a**	

Omit: Duplicates (1 + 2 + 6 = 1 + 6 + 2), Repeats (1 + 4 + 4). There are only three valid combinations: **a**, **b**, **c**.

Triangle apex's: Between **a–b**, 1 repeats;
Between **b–c**, 3 repeats; Between **c–a**, 2 repeats.
Put the remaining pool numbers in the remaining circles, where every side totals 9.
A unique solution!



Frame The Numbers

C: Add all possible numbers the grid can contain:
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45 (the number pool).
This sum must be equally divided among three rows, so that each row equals the same. Each row must total 15. Similar logic for columns and major diagonals makes each equal to 15.

Numbers 1, 2, 3 used; 4, 5, 6, 7, 8, 9 remaining (number pool).
Row1 has 3: 12 has to be placed in two cells, to total 15.
What combination of remaining (pool) numbers will give 12?
4 + 8; 5 + 7; 6 + 6 (fail, no duplicate numbers allowed).

A	3	B
1	C	D
E	F	2

If A = 5, B = 7: E = 9. Diagonal ECB ($\nearrow$) = 16, exceeds 15 without a number in C.

If A = 7, B = 5: E = 7 (fail, duplicate numbers).

8	3	**4**
1	**5**	**9**
6	**7**	2

If A = 4, B = 8: E = 10 (fail, only 1-9 allowed).

If A = 8, B = 4: E = 6, C = 5 (D = 9, F = 7).
Success, Unique Solution

Combination (5+7) tried before (4+8), proves a unique solution.

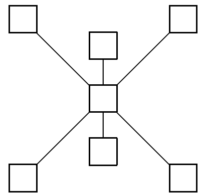
This is a same magic square 15, we encountered before.

Start the problem with *any* row or column, for a unique solution.

Another way of looking at this problem
Using the same logic, combine two pool numbers to get 14 (Col1 already contains 1). Only two factors: 5 + 9 and 6 + 8. One factor satisfies Row2, the other satisfies Col1 (1 from the problem figure is common to Row2 and Col1).
Row3, Col3, NW ($\nwarrow$) diagonal, need 13: 4 + 9, 5 + 8, 6 + 7 (three factors) satisfy Row3, Col3, $\nwarrow$ (2 is common to all three).

For sheer brute force (and sheer stupidity), you could try every possible number in every possible cell. You will prove a unique solution (if you can remember what numbers you tried in which cell, and what combinations remain to be tried).

F: Triskadekaphilia The grid has seven cells, and there are nine numbers. Two numbers must not be present on the grid! They are not two specific numbers. Different combinations exclude different numbers, at different times.



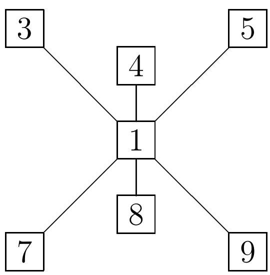
Pool numbers: 1–9. Ignore

- Duplicates (1 + 6 + 6 repeats/duplicates the number ‘6’),
- Repeats (1 + 3 + 9 = 1 + 9 + 3),
- Non-pool number (1 + 2 + 10; ‘10’ is not a pool number).

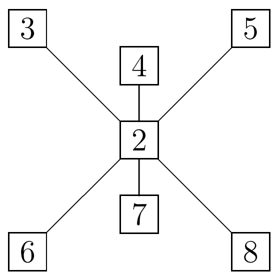
1 + 3 + 9 **a**	**1** + 4 + 8 **b**	**1** + 5 + 7 **c**	**2** + 3 + 8 **d**
2 + 4 + 7 **e**	**2** + 5 + 6 **f**	**3** + 4 + 6 **g**	

Do you feel a combination is missing from this list? Jot it down and sort out the numbers in increasing order, with the smallest number first. Then check if it is already listed. ‘3 + 1 + 9’ is a valid combination. Sorting, ‘139’ is the same as Combination **a**.

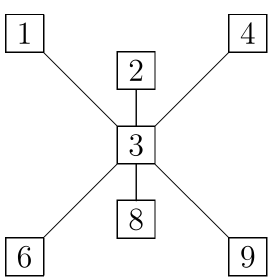
All three lines (column, both diagonals) share the central square. Identify three combinations that share the same number, There are *four distinct solutions*!



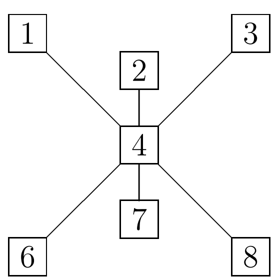
1: **a**, **b**, **c**



2: **d**, **e**, **f**



3: **a**, **d**, **g**



4: **b**, **e**, **g**

You looked up the word ‘triskadekaphilia: Obsession with the number thirteen’? Take a bow, problem solver!

A Counting Spree

E: Stated: 36 wheels; trucks need 6 tires, cars need 4 tires.

Implied, for a viable solution: Since there were trucks and cars, there must have been *at least one of each*.
There must be no 'spare' tires.

Assume the largest number of trucks and gradually decrease (or the largest number of cars, then decrease):

# of Trucks	Truck Tires	Tires Remaining	# of Cars	Spare Tires	Solution ✓ or ✗
6	36	0	0	0	✗
5	30	6	1	2	✗
4	24	12	3	0	*Solution 1*
3	18	18	4	2	✗
2	12	24	6	0	*Solution 2*
1	6	30	7	2	✗

6 trucks mean 0 tires remain for cars. This is illegal according to the rules of the problem (there must be at least one car).

Found both solutions? Excellent. But what does the problem *demand*? The *exact count* of cars and trucks at the signal!

Information provided by the problem statement is unconstrained and vague. You cannot say for certain, there could have been four trucks and three cars or two trucks and six cars. With two viable solutions, the question (posed by the problem) is unanswerable.

F: 36 wheels; trucks need 7 tires, cars need 5 tires.

# of Trucks	Truck Tires	Tires Remaining	# of Cars	Spare Tires	Solution ✓ or ✗
5	35	1	0	1	✗
4	28	8	1	3	✗
3	21	15	3	0	*Solution*
2	14	22	4	2	✗
1	7	29	5	4	✗

This problem has a unique solution, you can tell for certain that there were *exactly* 3 trucks and 3 cars at the signal.

Scissor Work

A: Cut the shape into four contiguous areas of:
1. Identical shape and size.
2. Identical size only.

Cut only along straight line cell boundaries.

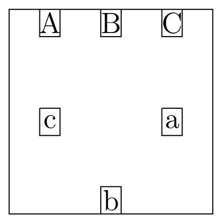
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Cut pieces cannot share (overlap) cells. All cells must be used (no discards). Each cut piece must form a contiguous area. Cells 1, 6, 11-15, do not form a contiguous area (single piece; common perimeter surrounding all cells). Think of cells as areas in an ice cube tray. These cell boundaries will fill three areas: Cells 1, 6, 11-15; instead of the single area needed.
Cells 8–7–11-10, 1–5–9–13… are valid areas.

Detailed analysis starts on page 10

B: Twisted Pathways

In the box, connect pairs of same letters using continuous (straight or crooked) lines that do not cross each other, or even touch. Lines cannot stray outside the bounding box.



Detailed analysis starts on page 12

Shapely Figures

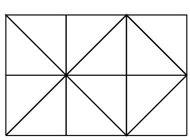
A, B, C, D: Four visible (obvious) primitive squares. One blended (not obvious) composite square (ABCD), comprising (and combining) all primitives.

A	B
C	D

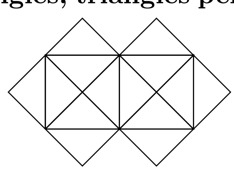
Any two adjacent squares form a rectangle, any three squares form an 'L' shape.

Figures below have visible basic (primitive) shapes and hidden (composite) shapes lurking within them. A primitive (simple) shape cannot be broken down (decomposed) into simpler ones. A composite shape comprises two or more primitive shapes.

Count the number of squares, rectangles, triangles per figure.



C

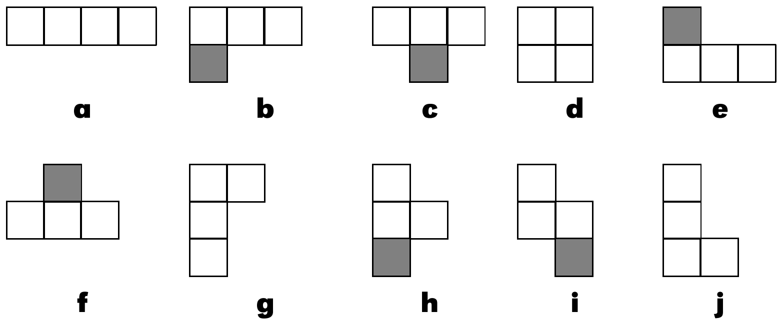


D

Detailed analysis starts on page 13

A: Four Square Four 16 cells must be cut along the cell edges, into four equal areas. Each area *must* contain exactly four cells.

In how many ways can four celled shapes be arranged?



In one row, in only one way (figure **a**).

In two rows, at least one cell per row (figures **b**, **c**, **d**, **e**, **f**):

- Three cells in Row 1, one cell in Row 2 (**b**, **c**)
- Two cells in Row 1, two cells in Row 2 (**d**)
- One cell in Row 1, three cells in Row 2 (**e**, **f**)

In three rows, at least one cell per row (figures **g**, **h**, **i**, **j**):

- Two, one, one cells in Row 1, Row 2, Row 3 (**g**)
- One, two, one cells in Row 1, Row 2, Row 3 (**h**, **i**)
- One, one, two cells in Row 1, Row 2, Row 3 (**j**)

In four rows, is a 90 ° rotated version of **a**.

To visualize the cell arrangements, keep cells in a row or column constant and move cells in the others. In **b**, three cells in Row 1 are kept constant, the cell from Row 2 (Col 1) is moved along Row 2 (to Col 2) to give **c**. Similarly visualize movement of the shaded cell from Col 1 to Col 2 of **h**, to give **i**.

Only figures (pieces) **a**, **b**, **c**, **d**, **i** are unique:
e rotated 180° and vertically flipped, is **b**.
g, **j** are versions of **b**.
f and **h** are 180° and 90° anticlockwise rotated versions of **c**.

Any arrangement of four contiguous cells *must* be one of these five pieces, rotated and/or flipped. Some mirrored/reflected/ equivalent pieces have been omitted. For example, after **c**, the next piece with three cells in Row1 and one cell in Row2 (below the third cell of Row1) is a vertically flipped version of **b**.

Piece **i** was learned from Solution 3 of the previous problem!

Try each unique piece in turn, to find solutions. First place one piece somewhere (easier to visualize piece placement at the edges of the composite shape). Then place other *same contour* pieces within the composite shape. Some pieces have no solution. Ignore rotations and reflections in the final figures, they are really the same solution (for example **k** and **l**, **m** and **n**).
Perhaps there are more solutions than those shown here.

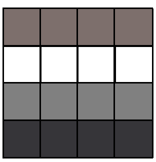
Identical shape and area figures use pieces/shapes **a**, **b**, **c**, **d**.
Shape **i** cannot form identical shape and area pieces.

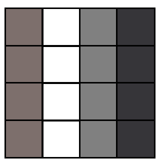
Solutions **k**, **m**, **o**, **p**, **q** have identical shape and area.

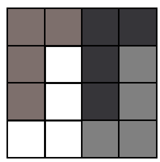
Solutions **r**–**z** are *some* possible identical area solutions.

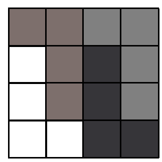
Solutions **w**, **x**, **y**, use shape **i**!

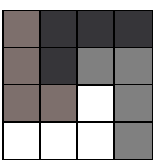
No identical area solution that contains four different shapes.

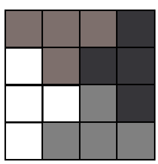

k

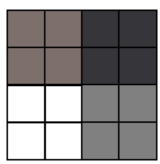

l

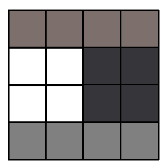

m

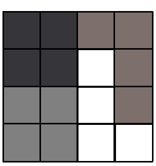

n

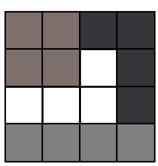

o

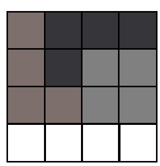

p

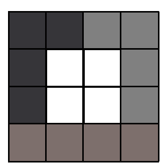

q

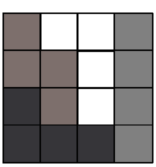

r

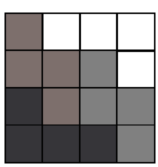

s

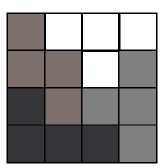

t

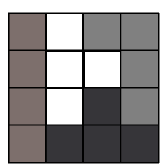

u


v


w


x

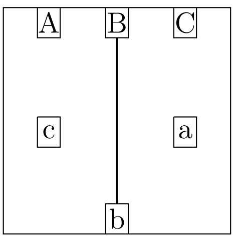

y


z

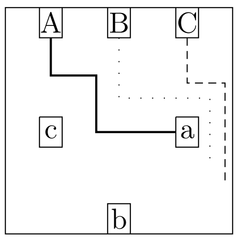
Pathing Of The Ways

While solving these path problems, *do not look at the final figure.* First draw the figure without any paths/connections. Then draw each path as it is explained (and you understand why it is drawn in that particular order). This will help visualize how paths can potentially cross (so you can examine methods to avoid this).

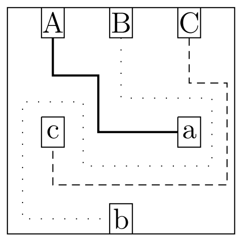
Try to connect each set (A-a, B-b, C-c…) first, in turn. Verify which connections lead to solutions, and which connections fail.
B1: Connect B-b first. A, c to its left and C, a to its right are permanently isolated on either side of B-b, can never connect.
B2: Connect A-a first. B and C have no choice in the path they must take (to the right of a). Now if B connects to b (B-b), C (to its right) can never connect to c (to its left).
B3: C-c must connect before B-b (B must go around, above, and to the left, of c). *Solution 1.*
B4: Connect C-c first. B-b must connect last (as b is on the edge of the puzzle, you cannot go around it). *Solution 2.*



B1



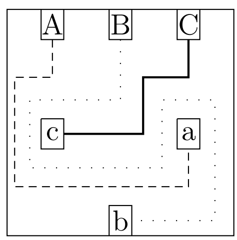
B2



B3

Connect A-a first. To the left of, below, and around c. This forces paths C-c then B-b, gives the solution shown in **B4**.

Connect C-c first. To the right of, and below a. This forces paths A-a then B-b, to give the solution shown in **B3**.

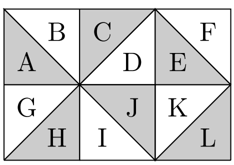


B4

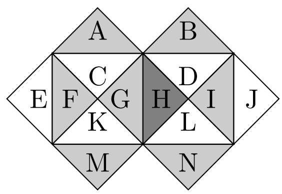
Counting Times

Primitive shapes combine to form composites. Labeling all primitive shapes or all vertices forming primitive shapes, or selective shading of primitives, aids visualizing.
Checking all possible shapes that can be made from each vertex in turn will make *hidden* shapes visible.
Always start with the smallest (primitive) shapes. Combine few primitives to form a composite shape, more primitives, to form a more complex composite shapes.
Try rotating the page (or inclining your head) to help recognize shapes in other orientations.
Only primitive shapes adjacent to each other (sharing exactly one common side) will form a composite shape.
Systematically search clockwise & anticlockwise, in four cardinal & four ordinal directions, left-to-right, right-to-left… to find/ identify hidden, blended, and composite shapes.

In both problems, all primitives are isosceles right triangles.



C



D

Triangles: **23**
Primitives: A-L
BC, DJ*, HI, AG, DE, EK, JK
AGHI, GABC, BCDJ, DJIH

Squares: **9**
AB, CD, EF, GH, IJ, KL
DEJK
ABCDJIHG, CDEFLKJI

Rectangles: **10**
ABCD, CDEF, GHIJ, IJKL
BAGH, CDJI, FEKL
ABCDEF, GHIJKL

ABCDEFGHIJKL

Triangles: **24**
Primitives: A-N
FC, CG, GK, KF, HD, DI, IL, LH
CGHD, KGHL

Squares: **11**
EF, AC, KM, GH, BD, LN, IJ
FCGK, HDIL
EFACGHKM, GHBDIJLN

Rectangles: **11**
ACEF, GHKM, BDGH, IJLN
EFKM, ACGH, GHLN, BDIJ
BDGHKM, ACGHLN
FCGKHDIL

*Triangle DJ repeats in squares ABCDJIHG, DJKE; count *once*.

The following discussion only concerns *isosceles right triangles*: Two primitives form a larger composite triangle if each primitive touches along the entire length of one side that is not the hypotenuse. Both hypotenuses touching along their entire lengths forms a square. In figure **C**: Primitive triangles B and C form a composite triangle; C and D form a square.

Two composite triangles (two primitives each) with one side each touching along an entire length (hypotenuses of two primitives!), form a four-composite triangle *if* both two-composite triangles are 90° to each other; form a rectangle for in-line two-composite triangles. In figure **C**: Composites BC and DJ touching sides C and D (hypotenuses of primitive triangles C and D!), form a four-composite triangle BCDJ; composites BC and DE touching sides C and D form a square.

Two squares touching each other along the entire length of a side, form a rectangle.

Two rectangles (made from two squares*) touching each other along the entire length of their longer side, form a composite square (really four smaller squares, eight triangles); touching each other along the entire length of their shorter side, form a longer composite rectangle.

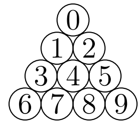
*The length of the rectangle's longer side is twice that of its shorter side.

In Summary:
Primitives forming a composite must be adjacent to each other.

Larger the regularly shaped composite (made from all similar isosceles right triangles, squares, rectangles), more the number of primitives it could be composed of.

Isosceles right triangles always have two sides of the same length, and 90° to each other. The third triangle side is the hypotenuse.
For (primitive and composite) isosceles right triangles only:
Two or four... adjacent primitives, *may* form larger triangles.
Two or four or eight... adjacent primitives, *may* form squares.
Four or six or eight or twelve... adjacent primitives, *may* form rectangles.

Round & Shiny



A: Turn the triangle upside down by moving three coins. Move not remove: Do not discard moved coins! They must move to a different position, and be part of the final figure.
All coins have identical size, shape, weight. They look, weigh, and feel, identical.
Compare the solution figures to the problem figures, *not* to other solution figures.

Detailed analysis starts on page 16

B: Domino Skeleton

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

Blank Grid

5 3	0 1	6 2	4 3
4 6	5 5	3 3	2 5
1 4	5 1	1 1	4 2
4 4	2 2	6 5	4 0
0 0	5 0	3 0	3 1
2 0	6 0	2 3	6 6
1 6	3 6	4 5	1 2

28 bones

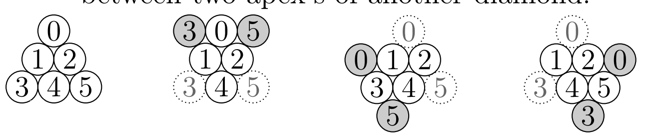
Each domino bone section contains a number between 0-6. Bone 14 (hereafter called 14) has sections 1 and 4; it is equivalent to 41, rotated by 180°. This bone will only fit a grid location that has 1 and 4 adjacent to each other in one of the cardinal directions N, S, E, or W. You cannot place a bone diagonally on the grid. Bone 14 *must* fit in one of these grid locations: BJBK (S), CJCK (S), EJEI (N), AOBO (E), EOEN (N), EODO (W).

To succeed at this puzzle (the goal), the entire surface of the blank grid must be completely covered with all the 28 bones. There should be no spare bones, no uncovered spaces on the grid, no overlapping bones, no bones partially off the grid.

Detailed analysis starts on page 19

A: Triangle from Any Angle! First try a simpler three row problem (figure below; use only coins 0-5). Moving two coins turns the triangle over.

- Viewing the triangle from three directions:
  - Triangle 1: Apex 0, Middle 12, Base 345.
  - Triangle 2: Apex 3, Middle 14, Base 025.
  - Triangle 3: Apex 5, Middle 24, Base 013.
- Are there any other shapes within the figure?
  - Small triangles: 012, 314, 524, **124**.
  - Diamonds! Each diamond has two apexes, 0142 (or 4201), 3421 (or 2134), 5214 (or 1452).
- Fitting a diamond into the larger triangle:
  - One diamond apex forms one triangle apex. Two coins protruding on either side of the other diamond apex, form the second and third apexes of the triangle. Move them to the first apex of the diamond!
- Triangle 012345 (triangle apexes 0, 3, 5) contains diamond 0142 (apexes 0, 4):
  - Coins 3, 5 (triangle apexes) are on either side of Coin 4 (diamond apex). Move 3, 5 to the other (triangle and diamond) apex 0. *Solution 1*
  - *Solution 2* and *Solution 3* are similar; move two coins between two apex's of another diamond.



Problem **Solution 1** **Solution 2** **Solution 3**

Coins 124 *never* move! Only the triangle apexes move.

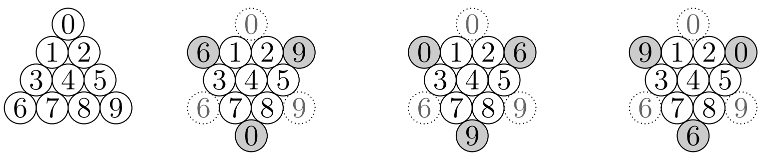
Optimal *Solution 1* is an 'in place' solution, where the coins remain in the same rows. *Solution 2* deletes the top row (Coin 0) and creates a new bottom row (Coin 5). *Solution 3* deletes the top row (Coin 0) and creates a new bottom row (Coin 3).

A ten coins (in four rows) solution moves three coins.

Analysis:
Row1 has one coin. Move (add) three coins to turn it into a four-coin row. This move uses up all the three coins without achieving the goal.

A seven-coin circular structure is formed with coins 1258734 (Coin 4 is in the center): Row2: 12 Row3: 345 Row4: 78 (with adjacent triangle apexes 6, 9).
Move these apexes to Row2. 0 from Row1 (top-most row) shifts to new Row5 (now the bottom-most row). *Solution 1*

Row4 formerly contained four coins, now contains two. Row2 formerly contained two coins, now contains four. Row3 coins are untouched and unmoved.



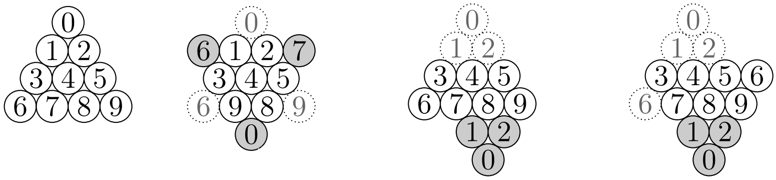
Problem **Solution 1** **Solution 2** **Solution 3**

Seven-coin circular structure 1234578 *never* moves! Only the triangle apex and outer coins of base (all triangle tips) move.

Problem: Base 6789, Apex 0; triangle apex points North.
Solution 1: Base 6129, Apex 0; triangle apex points South.
Solution 2: Base 0379, Apex 6; triangle apex points North-East.
Solution 3: Base 6850, Apex 9; triangle apex points North-West.

The directions and movements are relative. Since this is an equilateral triangle, each apex points to one of three directions (N, SE, SW for upright triangle; S, NE, NW for 180° rotated/ downward triangle). Depending on your perspective, the same triangle could point in all three directions simultaneously!

Now turn the triangle 180° around by moving four coins.



Problem **Cheat** **Move 012** **Solution**

You *could* move 67 on either side of 12; 9 where 7 was; 0 to a new row below 98. Instinctively you can see the 'cheat', the lack of elegance. The same result could be achieved by moving 69 on either side of 12 and 0 in a row by itself, below 98 (three coins).

Moving 4 (in the center of the structure) disrupts the overall shape of the structure. Disrupting, restoring the structure, and turning it upside-down with only four coin movements, seems difficult. So moving 0124 or 3467 or 7841 instinctively seems unwise. Similarly moving 0136 or 0259 makes a new structure with apex 2 or 1, and without the 180° rotation.

It is likely you moved the 'pointy' upper triangle apex downward, while trying to solve the original problem (turning the structure 180° around using three coins). 012 move below 678 or 789 (mirror locations). A last coin movement (6) gives the solution.

Coin 8 is the 'new' center of the inverted (180°, down) triangle.
Symmetrical solution: 7 is the center of the inverted triangle.
Two (upper) rows destroyed, two new (lower) rows created.

Hunt for (and discover) more *new* patterns in these structures. After all, playing the detective and sharpening your analysis skills is what you are cultivating via this book!

Domino Skeleton, Solution

Classifying, 'missing' bones are rotated existing bones!
Missing 10 (Col2) is equivalent to 01 (Col1).

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

0 0						
0 1	1 1					
0 2	1 2	2 2				
0 3	1 3	2 3	3 3			
0 4	1 4	2 4	3 4	4 4		
0 5	1 5	2 5	3 5	4 5	5 5	
0 6	1 6	2 6	3 6	4 6	5 6	6 6

These domino sections do not contain pips, they contain numbers 0–6. Each of the 28 bones can fit multiple possible grid locations.

Exhaustively, find *all possible* grid locations a given bone can fit:

1. Search the full grid line by line: left-to-right in rows and top-to-bottom in columns, to ensure no cell is missed out.
2. Search for the first section of the required bone.
3. Mentally put cross-hairs (✛) every time the first section is found on the grid. Search for its second (partner) section in four adjacent (cardinal) grid locations the cross-hairs point.
4. After finding one possible location for the second section in one cardinal direction, check the three remaining cardinal directions. Continue, search the complete grid for *all other possible locations* this bone can fit.
5. Search bone by bone (all sections from 0-6, in turn), find all possible grid locations each of the 28 bones can occupy.
6. Identify *unique* locations: The only (single) location that a particular bone can be placed in. Fix these bones on the grid.

For bones with both sections same, grid locations mirror and are equivalent (AJAK mirrors AKAJ); only list one grid location.

Given 28 bones and 56 cells/grid locations (equivalent to 56 section or 28 bone locations), there are exactly enough bones to blanket the entire grid.

Searching for bones: Invert numbers and consider the smaller. On the grid, BM forms bones South (50 = BMBN) and North (05 = BNBM). But since 05 < 50, finding bone-grid location combination 05 is equivalent to 50! Only list location BNBM.

At each stage list bones yet to be placed on the grid, to track them and prevent searching for mirrored bones (36 mirrors 63, only search for 36; 22 mirrors itself, list it in one direction only).

Bone	**All possible grid locations/orientations *each* bone can fit**
00	AJAK, AMAN, ANBN, BNCN
01	AJBJ, ANAO, FOEO
02	DLEL, CNCM
03	DLDM, CNCO
04	AKBK, AKAL, AMAL, BNBO (AK forms bones East & South)
05	DLDK, DLCL, AMBM, BNBM, FOFN (DL forms bones N & W)
06	AJAI, CNDN, FOGO
11	**BJCJ**
12	CJCI, EJFJ, HKGK, FLEL
13	HKHL, FLFK, FLGL, FLFM (FL forms a bone to N, E & S!)
14	BJBK, CJCK, EJEI, AOBO, EOEN, EODO
15	EJEK, HKHJ, GNFN
16	BJBI, CJDJ, EJDJ, GNGM, GNHN, GNGO
22	**CIDI**
23	GIFI, FJFI, FJFK, GKGL, GKFK, ELEM, CMDM
24	DIEI, BLBK, BLAL
25	GIHI, BLCL, BLBM, ELEK, CMCL, CMBM
26	CIBI, DIDJ, GIGJ, FJGJ, GKGJ
33	GLHL, DMEM, EMFM
34	FIEI, EMEN, CODO, COBO
35	FKEK, HLHM, FMFN
36	GLGM, DMDN, FMGM
44	**BKCK**
45	CKDK, CKCL, ENFN
46	ENDN, DODN, HOHN, HOGO
55	HIHJ, DKEK
56	HJGJ, DKDJ, HMHN, HMGM
66	**AIBI**

Placing bones with unique locations (highlighted **bold**) on the grid, makes matching the remaining bones and free grid locations easier, by shrinking the problem unknowns.

00 *must* fit AJAK (to prevent AJ being *orphaned*, and violating the rule of no uncovered grid spaces). Note: To cover an orphaned cell would require overlapping bones or a bone partially off the grid, another violation of the rules! Now, 04 cannot fit AKAL.

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

Placing a (domino) bone on the grid *may eliminate* possible grid locations of other bones; **manually check each location**.
Smart Way: Place a bone; eliminate other location references to it (if 66 = AIBI, cross out all other 'AI' and 'BI' in the table).
Rewrite *new* possible grid locations/orientations table.
Next, target bones with fewest possible grid locations.

Place 00. Eliminate impossible grid locations for bones.

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

01: ANAO, FOEO
02: DLEL, CNCM
03: DLDM, CNCO
04: AMAL, BNBO
05: DLDK, DLCL, AMBM, BNBM, FOFN
06: CNDN, FOGO
12: EJFJ, HKGK, FLEL
13: HKHL, FLFK, FLGL, FLFM
14: EJEI, AOBO, EOEN, EODO
15: EJEK, HKHJ, GNFN
16: EJDJ, GNGM, GNHN, GNGO
23: GIFI, FJFI, FJFK, GKGL, GKFK, ELEM, CMDM
24: **BLAL**
25: GIHI, ELEK, CMCL, CMBM
25 ≠ BLCL, BLBM as 24 = BLAL
26: GIGJ, FJGJ, GKGJ
33: GLHL, DMEM, EMFM
34: FIEI, EMEN, CODO, COBO
35: FKEK, HLHM, FMFN
36: GLGM, DMDN, FMGM
45: **ENFN**
46: DODN, HOHN, HOGO
46 ≠ ENDN as 45 =ENFN
55: HIHJ, DKEK
56: HJGJ, DKDJ, HMHN, HMGM

Now grid locations for 24 and 45 become unique.

Placing 24 = BLAL makes 04 = BNBO unique.
If 01 = ANAO and 05 = AMBM, AO and AM are not orphaned.
45 = ENFN eliminates many grid locations: FOFN, ENDN...

02: DLEL, CNCM
03: DLDM, CNCO
06: CNDN, FOGO
12: EJFJ, HKGK, FLEL
13: HKHL, FLFK, FLGL, FLFM
14: EJEI, EODO
15: EJEK, HKHJ
16: EJDJ, GNGM, GNHN, GNGO
23: GIFI, FJFI, FJFK, GKGL, GKFK, ELEM, CMDM
25: GIHI, ELEK, CMCL
26: GIGJ, FJGJ, GKGJ
33: GLHL, DMEM, EMFM
34: FIEI, CODO
35: FKEK, HLHM

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

36: GLGM, DMDN, FMGM
46: DODN, HOHN, HOGO
55: HIHJ, DKEK
56: HJGJ, DKDJ, HMHN,HMGM

If 02 = CNCM, *then* 05 = DLCL (CL not isolated), 34 = CODO (CO not isolated). *Then*, 36 = DMDN (DN not isolated).
But 05 = AMBM (exists), so 02 = DLEL (last grid location).

25 = CMCL (CL not isolated).
03 = CNCO, 06 = FOGO (both are now unique locations).
14 = EODO, 36 = DMDN (EO, DN not isolated).
33 = EMFM, 46 = HOHN (EM, HO not isolated).
16 = GNGM, 35 = HLHM (GN, HM not isolated).
34 = FIEI (unique location).

12: EJFJ, HKGK
13: FLFK, FLGL
15: EJEK, HKHJ
23: FJFK, GKGL, GKFK
26: GIGJ, FJGJ, GKGJ
55: HIHJ, DKEK
56: HJGJ, DKDJ

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

We highlight two (among several) paths to proceed:

- 12 = EJFJ? 56 = DKDJ, 35 = FKEK (DJ, EK not isolated). 35 = HLHM (exists), 12 ≠ EJFJ; 12 = HKGK? 13 = FLGL.
- 15 = HKHJ forces 25 = GIHI (HI not isolated). 25 = CMCL (exists), so 15 = EJEK, forces 56 = DKDJ.

Choose either path, get a unique solution!

An important pattern:
Assume 34 = EMEN. Then 33 cannot now be placed on locations DMEM, EMFM. 33 = GLHL.

There are far too many patterns to illustrate here. Plenty more interesting ones to ‘discover’ by yourself...

	A	B	C	D	E	F	G	H
I	6	6	2	2	4	3	2	5
J	0	1	1	6	1	2	6	5
K	0	4	4	5	5	3	2	1
L	4	2	5	0	2	1	3	3
M	0	5	2	3	3	3	6	5
N	0	0	0	6	4	5	1	6
O	1	4	3	4	1	0	6	4

A different decision earlier still leads to the same constrained conditions, many paths lead to this unique solution (if you play by the rules of the game).

What if no unique grid locations exist for any bone?
Assume Bone 1 (B1) can be placed on two possible grid locations L1 or L2; B2 can be placed on L3 or L4.
There are now *four* possible solution pathways (each starting with a separate bone-(grid) location combination):
B1-L1, B2-L3; B1-L1, B2-L4; B1-L2, B2-L3; B1-L2, B2-L4.

Which solution pathway would you start from? You will not know which one leads to a solution unless you follow (try) all four solution pathways until their end. Only then will you know for certain if the grid supports a unique or multiple solutions. Along the way, each pathway may branch out into more possibilities. A potentially daunting task ahead, but manageable. Correct?

To whittle down possible solution pathways, check if placing a bone on a particular grid location eliminates other possible grid locations of other bones. This reduces the problem domain. The aim is to find a pathway (or pathways) with unique bone-location combinations, and hope it is the correct solution pathway (or pathways, if multiple solutions exist).

List all grid locations/orientations each bone can fit.

Table on page 24 has no unique bone/grid-location combination (starting pathway) for this grid. Fortunately, a unique solution pathway exists.

	A	B	C	D	E	F	G	H
I	2	3	4	1	3	3	3	1
J	6	6	3	2	5	0	5	5
K	4	5	0	0	6	4	1	1
L	6	5	3	1	6	4	0	6
M	6	0	0	2	3	4	1	5
N	4	5	3	2	0	1	1	6
O	4	0	2	2	4	2	5	2

Bone	**All possible grid locations/orientations *each* bone can fit**
00	CKDK, BMCM
01	DKDL, GLGK, GLGM, ENFN
02	DKDJ, CMDM, ENDN, BOCO
03	FJFI, CKCJ, CKCL, CMCL, CMCN, ENEM
04	FJFK, GLFL, ENEO, BOAO
05	FJGJ, FJEJ, CKBK, BMBL, BMBN, BOBN
06	DKEK, GLHL, BMAM
11	GKHK, GMGN, FNGN
12	DIDJ, DLDM, FNFO
13	DIEI, HIGI, DLCL
14	DICI, GKFK, GMFM, FNFM
15	HIHJ, GKGJ, HKHJ, GMHM, GNGO
16	HKHL, DLEL, GNHN
22	DMDN, DNDO, CODO
23	AIBI, DJCJ, DMEM, DNCN
24	DOEO, FOEO
25	DJEJ, FOGO, HOGO
26	AIAJ, HOHN
33	EIFI, FIGI
34	BICI, CJCI, EMFM
35	EIEJ, GIGJ, CLBL, CNBN
36	BIBJ, CJBJ, EMEL
44	FKFL, FLFM, ANAO
45	AKBK, ANBN
46	AKAJ, AKAL, FKEK, FLEL, ANAM
55	GJHJ, BKBL
56	EJEK, BKBJ, BLAL, HMHL, HMHN
66	AJBJ, EKEL, ALAM

Got all the possible grid locations and orientations of each bone? Noticed 23 was missing COCN in the list? Very sharp of you! After all, you *are* following the rules of the pathway.

No unique bone-location combination for this grid, target fewest: Candidate bones 00, 24, 26, 33, 45, 55, have two combinations each. A good strategy would check if placing a candidate bone on a specific grid location makes placing any other bone (preferably a candidate bone) on the grid impossible. Since all bones must must be placed on the grid, discard that specific bone-location combination. For example, assume candidate bone B1 is put on grid location L1. *Now*, placing bone B2 on the grid becomes impossible. Discard grid location/orientation B1-L1 (since B2 *must* find its place on this grid).

Checking, no candidate bone gives a 'unique' bone-location fit.
A multi-pronged starting pathway!

Look for starting pathways that constrict or restrict other possible bone/grid-location combinations. Go down each solution pathway, see if it leads to a solution.

	A	B	C	D	E	F	G	H
I	2	3	4	1	3	3	3	1
J	6	6	3	2	5	0	5	5
K	4	5	0	0	6	4	1	1
L	6	5	3	1	6	4	0	6
M	6	0	0	2	3	4	1	5
N	4	5	3	2	0	1	1	6
O	4	0	2	2	4	2	5	2

Constriction at the grid corners: Corner cells have only two directions of bone formation. AO has AN (North), BO (East).

Less restriction at the grid edges: Edge cells have three directions of bone formation. EO has EN (North), FO (East), DO (West).

Freedom inside the grid: Here freedom is *bad*, it gives four directions of bone formation. GN has GM (North), HN (East), GO (South), FN (West).

If 55 = GJHJ: Three locations for 15 become impossible; two candidates (GMHM, GNGO) still remain. 13 = HIGI prevents orphaning HI. 33 = EIFI (last location remaining, for 33).

If 33 = FIGI, 15 = HIHJ, 55 = BKBL, 45 = ANBN, 04 = BOAO. Cell GJ: North (GI) and East (HJ) cells are occupied. South cell (GK = 1) cannot be used as 15 = HIHJ already. So, 05 = FJGJ!

If 45 = ANBN, 04 = BOAO. Still two locations for 55!

If 15 = GNGO, 26 = HOHN. AI must form a bone: South (now impossible as 26 = HOHN); *must* form a bone East (23 = AIBI).

By the same logic, *if* 26 = AIAJ, 25 = HOGO. Now *assume* 11 ≠ GKHK. Then 11 = GMGN or 11 = FNGN. Either way, 56 = HMHN (to prevent isolating HN).

If 04 = GLFL or 46 = FLEL, 44 = ANAO (last possible location for this bone, on this grid).

If 11 = GKHK, you cannot place a bone on these grid locations: 15 = GKGJ, 15 = HKHJ, 16 = HKHL, 01 = GLKL, 14 = GKFK.

If 44 = FKFL, 46 ≠ FKEK, 46 ≠ FLEL, 04 ≠ FJFK, 04 ≠ GLFL, 14 ≠ GKFK, 44 ≠ FLFM (duplicate bones not allowed). *Now*, corner cell AO cannot form a bone to the North (44 = FKFL already exists), must form a bone to the East (04 = BOAO).

Candidate bones 45 & 55 overlap, two pathways* to choose:
If 45 = AKBK, 55 = GJHJ, 13 = HIGI.
If 55 = BKBL, 45 = ANBN, 04 = BOAO. Referring to HJ, 55 ≠ GJHJ (since 55 = BKBL); 15 *must* be located on HIHJ or HKHJ. Now, all other grid locations for 15 (GKGJ, GMHM, GNGO) become impossible.

*Solution pathways are valid subject to '*If*' condition being true. The complete solution is given overleaf.

A word to the wise:
As more bones are fixed at their final grid locations, there are fewer possible locations for the remaining bones to fit, on this grid. The problem domain shrinks, finding solutions is easier.
Each bone occupies two (even number) adjacent grid locations. If at any time you have an odd number of empty grid locations, abandon hope! At least one grid location will remain uncovered.
Complexity increases with a larger grid (more grid locations to be covered), with more domino bones (to fit the large grid), with a larger number of grid locations each bone can be placed in, with no unique bone/grid location combination.
If at the start no bone can be placed in a unique grid location, start with the bone that has the least possible grid locations.

Play to your strengths
Know your weaknesses and either convert them into strengths (here, look extra carefully, in ways you normally neglect or are careless), or reduce the weakness (double-check).
Patterns like 11, 22... are easy to locate. You may have trouble realizing that 01 and 10 are the same bone. Do not locate bone 10 on the grid (its equivalent 01 will have already been located).
Some people (especially the left-handed) are comfortable reading right-to-left. Others read well from bottom-to-top.
On recognizing a pattern, it is easy to break focus and neglect others. Similar patterns involving the same bone sections could be missed. You view what you want to view, ignoring what you consider trivial or inconsequential. Book I has many examples of this, in Chapter 1. Stick to the formula, look carefully, follow the rules outlined on page 19, do not jump the gun.
Locating: 05 = BMBL, then BMBN. 25 = FOGO, then HOGO! Typically, grid locations at extreme corners are neglected.

There will be other patterns, if you look hard enough. Normal is boring! Take the more scenic (and instructive) pathways, make mistakes and learn on the journey. All the best!

This is the solution.

Interchangeable start pathways!
34 = CJCI and 36 = BIBJ are also possible.
33 = FIGI, 35 = EIEJ, 05 = FJGJ, are also possible.
Start with these interchangeable pathways, and you get slightly different solutions!

	A	B	C	D	E	F	G	H
I	2	3	4	1	3	3	3	1
J	6	6	3	2	5	0	5	5
K	4	5	0	0	6	4	1	1
L	6	5	3	1	6	4	0	6
M	6	0	0	2	3	4	1	5
N	4	5	3	2	0	1	1	6
O	4	0	2	2	4	2	5	2

Bone sections need *distinct symbols*: images, numbers, letters...
This puzzle needs any seven distinct symbols (currently has 0–6).

0.1 Blurred Vision

Your brain *must be* spinning, with shapes squirming and dancing in front of your eyes. But with 'shapes' behind you, visualizing and using shapes in the next chapter should be a breeze.

This chapter develops visual thinking and spatial reasoning. The game of Tetris (or its many clones) and playing with actual (physical) Lego blocks will also enhance spatial ability, and the understanding of shapes in two and three dimensions.

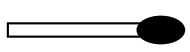
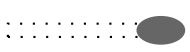
Time spent in the 'analysis' phase of problem solving is critical. Invest little time and get no solution, or a half-baked solution, or do not get all possible solutions (and have no way of knowing if you have chosen the 'optimum' solution). Invest too much time analyzing and you may never reach your goal (the solution)! Strike a *timely* balance, to ensure you get the best of both worlds. While learning, time is 'wasted' making mistakes as you stumble along. After understanding fundamental ideas and concepts (of shapes, mathematics, logic...), 'analysis' time will decrease!

In most problem, solutions will not come instantly and easily. No doubt you realize this by now (if not way back). Ruminate over a situation, be prepared to invest time, be determined to analyze all possible ways to proceed forward, follow through to all possible successes. Solutions will come, they just require your willingness to give them *time and effort.*

Playing With Fire

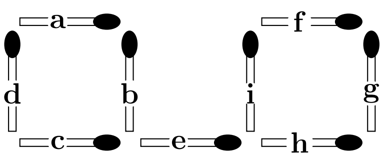
All match problems and their solutions are in two dimensions (2D), matches lie on a flat surface. Breaking matches, placing matches atop each other (overlapping) or duplicating (matches touching along their partial or entire length) so as to count as a single match, loose ends (matches present but not part of the final figure or shape i.e. ‘orphans’), are forbidden.
Removing or discarding a match: It is not part of the final figure.
Moving or rearranging a match: It must be present in (and part of) the final figure, though in a different location/position.

Legend for all match problems:

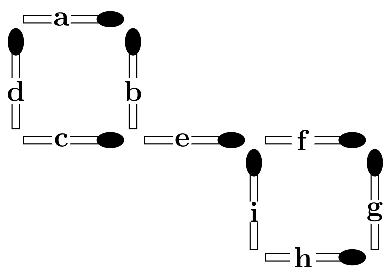
 Matches from the problem (do not move in the solution)
 Matches that have been moved or removed
 Matches in new positions, in the solution

For identification and reference, matches are labeled with letters, enclosed spaces (squares, triangles, etc.) with numbers.
A *group* of matches and enclosed spaces (squares, triangles) are not separated with commas, but are enclosed in square brackets. For example, three matches [cfh], two squares [47], [g], [3]. . .

Do not mentally visualize these problems and their solutions, invest in a matchbox! Analysis gets confusing if you move around matches in your imagination.



Match [e] does not have an opposite match.

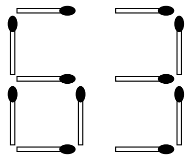


Match [e] is missing an opposite and an adjacent match.

Primitive (made from four matches) squares always have the same size and (obviously) shape. Each match must form an enclosed square shape, with three other matches (two adjacent, one opposite, *four matches form a square*). [e] violates this rule for both the figures shown above, and is *orphaned* (a loose end)!

F: Change The Total

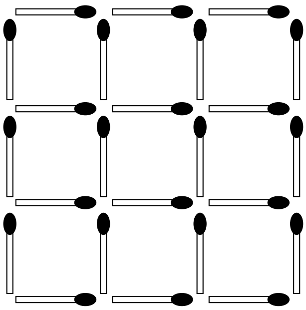
Eleven matches are arranged to form the digital number 63. Move *only one* match to another position to form a different two-digit digital number.



Detailed analysis starts on page 30

Don't Fence Me In

All these problems refer to the same problem figure!



All squares *must* have the same size and shape.

Keep 6 squares, remove matches: **G:** Four **H:** Five

Keep 7 squares, remove matches: **L:** Two

Keep 4 squares, remove matches: **M:** Eight

Detailed analysis starts on page 38

All squares *must* have same shapes, but *may have different sizes.*

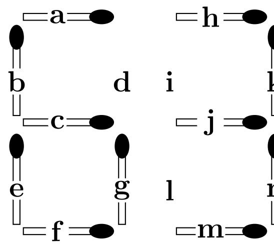
A: Remove 8, leave 6 squares **B:** Remove 8, leave 4 squares

Detailed analysis starts on page 44

C: Remove the least amount of matches, to *destroy all squares.* And yes, matches will end up orphaned.

Detailed analysis starts on page 51

F: A Total Change!



Were you biased into thinking the required number should be bigger, or that you had to move a match within the same digit?
Only seven possible match locations, a–g or h–n, form a *digital* number.
Transform *only one* match.

There are two types of transformations:
Displace matches within the same digit OR *Move* matches from one digit, *Add* it to the other (*Move* and *Add* are different aspects of the same kind of transformation).

Some digital numbers cannot be created:
More than one match would be *out of place or extra*,
AND/OR
One or more locations would need to be filled.

For first digit 6 (6 already exists):
1 [*abcef*], [d]; 2 [*bg*], [d]; 3 [*be*], [d]; 4 [*aef*], [d]; 7 [*bcef*], [d].

Create 0: displace [c] to location d.
Create 5: move [e] to second digit 3.
Create 8: move one match from second digit 3 to location d*.
Create 9: displace [e] to location d.

For second digit 3 (3 already exists):
0 [*j*], [il]; 1 [*hjm*]; 4 [*hm*], [i]; 6 [*k*], [il]; 7 [*jm*]; 8 [il].

Create 2: displace [n] to location l.
Create 5: displace [k] to location i
Create 9: move one match from first digit 6 to location i.

First digit 6 can transform to 0, 5, 8, or 9.
Second digit 3 can transform to 2, 5, or 9.

Try all *possible* combinations in turn, to see which become valid digital numbers by transforming a single match:

02 03 05 09 52 53 55 59 62 63
65 69 82 83 85 89 92 93 95 99.

(Underlined digits do not change from problem to solution).

Currently pause this analysis pathway, seek a more efficient one.

[e] is the only match that can be transformed and still have the first digit remain a valid digital number (5). Displacing it to location d gives 93; moving it to location i gives 59. Moving it to location l gives ∂, an invalid digital number.

Transforming any other match creates invalid digital numbers. The transformed match cannot be moved to the other digit, it must be displaced within the digit, to recreate a valid digital number. For example, displace [n] to location l (3 becomes 2).

Only three possible vacant locations to move a match: d, i, l.
Transform each match to one of these three locations. From the results, only consider valid digital numbers.
[c] *must* displace to location d (03).
[e] can displace to location d (93) or move to location i (59).
[n] *must* displace to location l (62).
[k] *must* displace to location i (65).
No other *single* match transformation creates digital numbers!
You could argue that 03 is really 3, but still it *is* a number.

59 is the only number where both the digits transform. [e] *moves* from the first digit to the second digit, location i.
53 is not a solution, [e] has not yet been placed.
63 is the problem figure.

Continue, analyze all possible combinations (paused previously).
If the first digit transforms to 0: One match ([c] → d) has already moved, the second digit 3 *must remain as it was.*
02, 05, 09, become impossible!

If the first digit transforms to 5: [e] must move to location i on the second digit, for it to continue remaining a (new) digital number 9. [e] at location l is not a digital number. If the first digit transforms to 5, the second digit *must transform to* 9.
52, 53, 55, become impossible!

If the first digit remains 6: Second digit must transform using a match *within the digit itself* (match displacement).
62 (move [n] to location l) or 65 (move [k] to location i) are the only two possible digital numbers.
63 is the problem figure, where *no matches have moved.*
69 is impossible (3 needs an *extra match* added to location i, to transform to 9).

*Page 30 hinted at transforming the first digit (6) into 8. This needed a match in location d (added from the second digit, 3). This is *impossible*! Move any match from 3, and it ceases to be a digital number, ceases to be 'stable'.
82, 83, 85, 89, all become impossible!

If the first digit transforms to 9: [e] must move to location d. One match has moved, the second digit 3 must remain as it was.
92, 95, 99, become impossible!

All invalid number combinations are impossible because of one or more of these conditions:

- They need one match in each digit to transform. For example, 02 needs [c] displaced to location d, and [n] displaced to location l.
- Extra matches are needed:
  (Non-existent) Twelfth match at location i for 09, 69, 99; at location d for 82, 83 85.
  (Non-existent) Twelfth and thirteenth matches at locations d and i for 89, at locations i and l for 68...
- A match is extra (only ten matches needed): 52, 53, 55 do not need [e].

If you were observant enough to notice this font for 7 has four segments (equivalent to locations badg) instead of three (adg) present in real 'seven segment' digital displays, good for you!

Most people look for easy and instinctive solutions like 93 or 59. Did you use problem solving analysis to get all five solutions?

If the problem allowed transforming *one or two* matches, there would be more solutions for the (63, eleven match) problem.
Transform one match: 03, 59, 62, 65, 93 (already found).
Transform two matches: Nine more solutions,
02, 05, 29, 39, 50, 56, 84, 92, 95.
Fail, needs a twelfth match: 09, 58, 69, 82, 83, 85, 99.
Fail, needs a thirteenth match: 89.
Fail, only ten matches needed: 52, 53, 55.

Two match transformation solutions are explained overleaf.

With two matches, the first digit 6 can transform into:
0: Displace [c] to location d.
2: Displace [b] to location d; move [g] to 3.
3: Displace [b] to location d; move [e] to 3.
5: Move [e] to 3.
6: Both match *displacements* take place in 3.
8: Borrow a match from 3.
9: Displace [e] to location d.

With two matches, the second digit 3 can transform into:
0: Displace [j] to location i, borrow a match from 6.
2: Displace [n] to location l.
3: Both match *displacements* take place in 6.
4: Displace [h] to location i, move [m] to 6.
5: Displace [k] to location i.
6: Displace [k] to location i, borrow a match from 6 (first digit!).
7: Move [jm] to 6 (impossible, 6 can only accept one match).
8: Borrow two matches from 6 (impossible, 6 can only lend one).
9: Borrow a match from 6.

Try all possible combinations in turn. Only accept valid digital numbers involving two transformations. Get nine solutions.

An unfettered imagination will extend this puzzle to more combinations of digits and different amounts of matches to be moved. Transform all numbers from 00 – 99, if you have the patience!

Borrowing And Lending Depending on the number of match transformations (one or two) allowed, digits could be

Lean: Cannot lend matches and still remain a digital number. Can borrow matches from the other digit, and get fat.

Fat: Lend matches to the other digit (and become lean) *or* borrow matches from the other digit, and get fatter.

All matches moved from one digit must be lent to the other digit (which must borrow *all* lent matches; *discarding matches is not an option*). Solutions moving two matches: 29, 39, 50, 56, 84.

Neither a borrower nor a lender be! Displacing (rearranging) matches within digits implies no match exchange (borrowing or lending) between digits. Solutions moving two matches: 02, 05, 92, 95.

Borrowing and lending, using one or two match transformations:

0 is formed using six matches

1: Impossible, more than two match transformations needed.
2: Displace [b] → location c. Lend [g].
3: Displace [b] → location c. Lend [e].
4: Impossible, more than two match transformations needed.
5: Displace [d] → location c. Lend [e].
6: Displace [d] → location c.
7: Impossible, more than two match transformations needed.
8: Borrow one match for location c.
9: Displace [e] → location c.

Ignore digit transformations that need more than two matches,

1 is formed using two matches

A digital number formed using the least amount of matches (two), is the *leanest digital number*; can never lend matches, must borrow matches to transform! *Implied information*: A valid digital number must contain at least two matches.

4: Borrow two matches for locations b, c.
7: Borrow one match for location a.

2 is formed using five matches

0: Displace [c] → location b. Borrow one match for location g.
3: Displace [e] → location g.
5: Displace [d] → location b, [e] → location g.
6: Displace [d] → location b. Borrow one match for location g.
8: Borrow two matches for locations b, g.
9: Displace [e] → location g. Borrow one match for location b.

3 is formed using five matches

0: Displace [c] → location b. Borrow one match for location e.
2: Displace [g] → location e.
4: Displace [a] → location b. Lend [f].
5: Displace [d] → location b.
6: Displace [d] → location b. Borrow one match for location e.
7: Lend [cf].
8: Borrow two matches for locations b, e.
9: Borrow one match for location b.

4 is formed using four matches

1: Lend [bc].
3: Displace [b] → location a. Borrow one match for location f.
5: Displace [d] → location a. Borrow one match for location f.
7: Displace [b] → location a. Lend [c].
9: Borrow two matches for locations a, f.

5 is formed using five matches

0: Displace [c] → location d. Borrow one match for location e.
2: Displace [b] → location d, [g] → location e.
3: Displace [b] → location d.
4: Displace [a] → location d. Lend [f].
6: Borrow one match for location e.
8: Borrow two matches for locations d, e.
9: Borrow one match for location d.

6 is formed using six matches

0: Displace [c] → location d.
2: Displace [b] → location d. Lend [g].
3: Displace [b] → location d. Lend [e].
5: Lend [e].
8: Borrow one match for location d.
9: Displace [e] → location d.

7 is formed using three matches

1: Lend [a].
3: Borrow two matches for locations c, f.
4: Displace [a] → location b. Borrow one match for location c.

8 is formed using seven matches

The maximum (all seven) matches are used, is the *fattest* digit! A digital number cannot have more than seven matches.
Implied information: 8 cannot borrow matches, must 'shed' (lend) matches for transforming to other digits.

0: Lend [c].	2: Lend [bg].	3: Lend [be].
5: Lend [de].	6: Lend [d].	9: Lend [e].

8 can shed a minimum of one match (for 0, 6, 9), a maximum of five matches (for 1; but this is impossible if you are only allowed two match transformations).

9 is formed using six matches

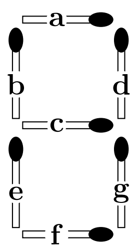
0: Displace [c] → location e.
2: Displace [g] → location e. Lend [b].
3: Lend [b].
4: Lend [af].
5: Lend [d].
6: Displace [d] → location e.
8: Borrow one match for location e.

Any digital number can transform into any other digital number, given enough match transformations. Our examples allow digital number transformations using one or two matches.

Two match transformations can interchange for the same results. To transform 6 to 2:
Displace [b] → location d. Lend [g].
OR
Displace [g] → location d. Lend [b].

Time To Pause & Reflect

A digital clock is called a seven segment display (and now you know why). Each digit has seven segments, all of which can potentially light up. A special combination (a pattern!) of segments lighting up determines the number displayed:



0: [abdefg]	1: [dg]	2: [acdef]
3: [acdfg]	4: [bcdg]	5: [abcfg]
6: [abcefg]	7: [adg]	8: [abcdefg]
9: [abcdfg]		

Capital letters Q and R cannot be represented, but small letters q ([abcdg]) and r([cef]) can!

Digital numbers are constructed exclusively from horizontal and vertical lines (no diagonal lines or curves). The original seven segment display was only meant to display number patterns. However, you can light up any combination of segments and either get letter patterns (A = [abcdeg], b = [bcefg]. . .) or nonsense ([dcef]). Letters with slanting lines (K, M, N, V, W, X, Y, Z) cannot be represented, some letters cannot be uniquely or unambiguously represented (how would you distinguish between letters 'O', 'D', and the number 0?).

Looking hard enough, you start seeing all sorts of patterns:
Rotate 4 by 180° and it looks like the letter 'h'.
3 rotated by 180° looks like the letter 'E', when rotated by 90° clockwise looks like the letter 'w', when rotated by 90° anticlockwise looks like the letter 'm'!
8 is two squares (shapes) stuck together, 0 is a rectangle!

Start noticing the shapes these matchstick puzzles form. This chapter will test practical application to your 'shapely' learning.

A digital number is constructed using a combination of seven matches. All seven match segments in place create 8. As a shape, 8 has two squares sharing a common match ([c]). Each match that is part of a primitive square needs two adjacent matches and one opposite match ([e] needs adjacent [cf] and opposite [g] *supporting* matches).
6: [abc] are not a square (has only three matches, you need four).
[a] and [c] are missing an adjacent match (which should have been in location d), [b] is missing an opposite match (location d).
[ab] (both matches) are *orphaned.*
[c] belongs to the complete square [cefg] and the partial square [abc] (technically not a square). Although its quota is incomplete with [ab], it is stable due to [efg]. [c] is *not orphaned.*

Structure Stability: Removing a shared (common) match destroys two squares! It may also create orphans. Removing [c] in the structure 8 breaks both squares, and also orphans all the remaining matches.

Removing a match that is unique to a square (obviously) destroys only one square. For the structure 8, removing [a] orphans [bd]. [abd] form a *set*, are *dependent* on each other. Removing any of them necessitates removing all the others, to maintain structure stability (prevent orphans). [c] loses one square, but is not orphaned as it is still a part of, and belongs to, the other (stable, intact) square structure [cefg].

Later examples will show how to remove some (independent) matches, without affecting stability of the remaining structure (without creating orphans).

Fencing Lessons

A square must have four sides. An isolated square needs four matches, two isolated squares need eight matches…

Two connected (adjacent, therefore dependent) squares in a row or column need seven (not eight) matches. The first square always needs four matches; all subsequent squares always need three more matches!

For a grid of squares, the situation is different. If three squares are connected in an 'L' shape, with one square sharing two of its adjacent matches, creating a fourth square needs only two more matches!

Identify which square(s) each match belongs to (forms a stable four-side structure with):

Outer (external) match forms the border/outline/enclosure/periphery, of the overall structure (shape), belongs to one square, and relies on a set of three other matches for stability of that square (a match in a triangle relies on two other matches).

Inner (internal, within the overall structure) match is always shared by (belongs to, depends on) two adjacent squares; relies on either of two sets (of three matches each) for its stability. A triangle match relies on *either of* two sets of two matches each.

Breaking squares: Removing a match may create structure instability (orphans). Remove all orphans (restore stability).

Removing an outer (belongs to one square) match breaks one square. Removing an inner (shared by exactly two adjacent squares) match breaks two squares. While removing matches, do not break the dependency on both these squares.

Each remaining match must form a stable square structure with

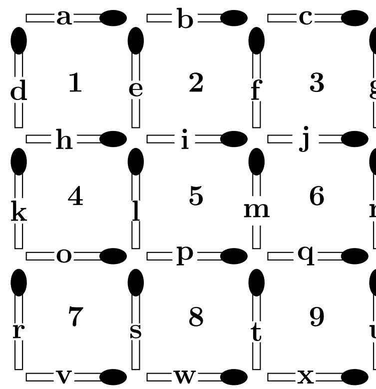
One set of three matches if it is an outer match, *or*

Two sets of three matches belonging to two adjacent squares, if it is an inner match.

Remember, a single match can break/destroy:

- One square, it must be a (non-shared) outer match.
- Two squares, it must be shared between exactly two adjacent squares (therefore must be an inner match).

A single match cannot destroy more than two squares.



Per square, identify matches that can be removed and still maintain a stable (no orphans) structure.

Remove [a]: Orphaned [d] must also be removed ([ad] are a set). [1] is broken. [e] belongs to [1] and [2]. As long as [2] is not broken also (do not remove any of [bif]), [e] is not orphaned, the structure remains stable.
Similarly, [h] belongs to [1] and [4] (adjacent squares sharing [h]). Now that [1] is broken (by removing [ad]), do not break [4] as well (do not remove any of [kol]).

Remove [b]: [2] is broken. [e] still belongs to [1], [f] still belongs to [3], [i] still belongs to [5].

Remove [i]: [25] are broken (by shared [i]); remove orphan [b]. [e] (still belongs to [1]) and [f] (still belongs to [3]) are not orphaned. Similarly, [m] still belongs to [6], [p] still belongs to [8], [l] still belongs to [4].

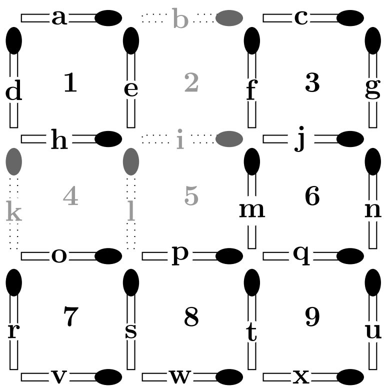
If you remove [i], you must remove [b] ([i] is dependent on [b]). Remove [b] without affecting [i] ([b] is independent of [i]).

Remove [e]: [12] are broken. Remove [abd] to prevent orphans.

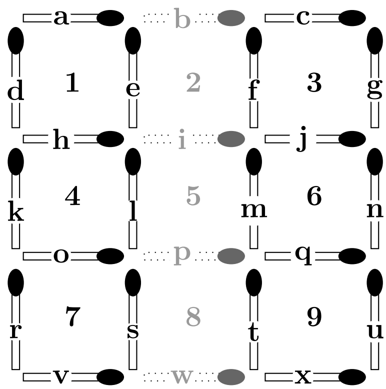
Remove [ef]: [123] are broken ([12] share [e], [23] share [f]). Remove unstable [abcdg], and prevent orphans.

All other matches are symmetrical versions of these.
Only three distinct squares and their symmetrical versions:
[1]: [3], [7], [9]; [2]: [4], [6], [8]; [5].

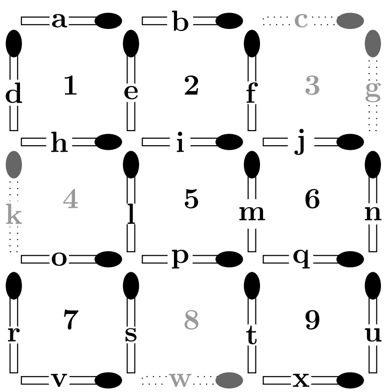
For a symmetrical figure, solutions *probably* involve removing matches symmetrically.
Try removing combinations and sets, of matches. Solutions will come with trial and analysis, not error or randomness!
Fair warning, analysis will take time and yield *many solutions*. There may be more solutions than those shown, keep analyzing!



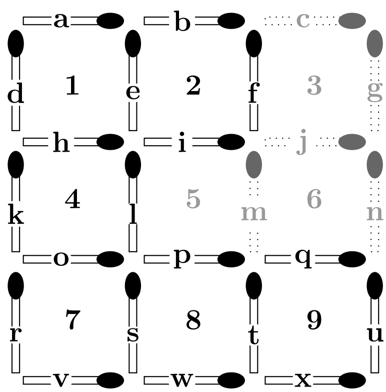
G1



G2



G3



H1

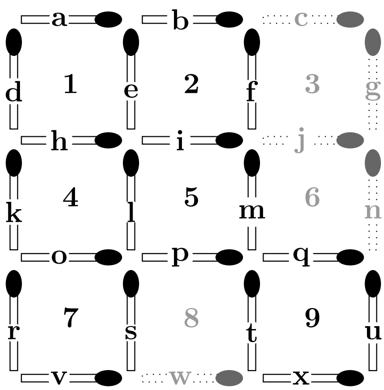
G: Breaking a square needs at least one match. A single match shared between two adjacent squares (therefore an inner match!) can break both squares. Remember to eliminate orphans.

Only two combinations break three squares with four matches:

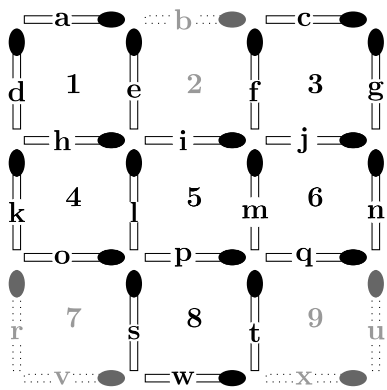
- Two matches break two squares (one square per match), the other two matches break the third square.
- One (shared) match breaks two adjacent squares, the other three matches break the third square.

Remember, no orphans are tolerated.

G1: [b] eliminates [2], [i] eliminates [5], [kl] eliminates [4].
or, [b] eliminates [2], [k] eliminates [4], [il] eliminates [5].
or, [i] eliminates [25] (remove orphan [b]), [kl] eliminates [4].
G2: [b] eliminates [2], [w] eliminates [8], [ip] eliminates [5].
G3: [k] eliminates [4], [w] eliminates [8], [cg] eliminates [3].



H2



H3

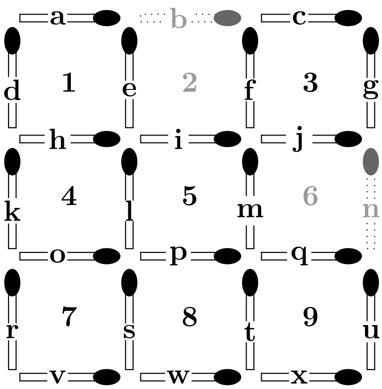
H: *Three combinations* of five matches break three squares:

- One match each breaks two squares. The third, fourth, fifth matches break the third square.
- Two matches each break two squares. The last (fifth) match breaks the third square.
- One (shared) match breaks two adjacent squares. Four more matches break one more square.

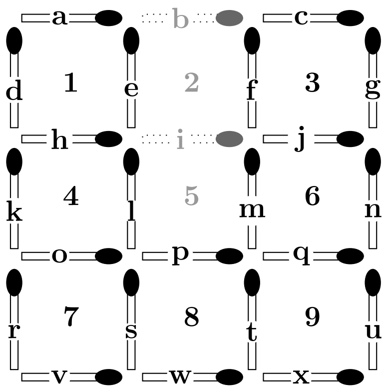
The *order* in which matches eliminate squares determines which combination is used. Ensure stability (avoid orphans) as each square is eliminated, in turn:

H1: [n] destroys [6], [m] destroys [5], [cgj] destroys [3],
or [cg] destroys [3], [jn] destroys [6], [m] destroys [5].
or [m] destroys [56] (remove orphan [n]), [cgj] destroys [3].

Matchstick puzzles use visual shapes. Transforming (moving or removing) a match changes or 'morphs' the overall shape/ contour of the structure. The old (original structure) analysis is no longer relevant (to this new structure).



L1



L2

L1 is obvious, **L2** is interesting. Did you get both?

In **L1**, you can remove the two matches in any order.

In **L2**, only after destroying [2] can you destroy [5]!
More interesting After removing outer match [b] (to destroy [2]), inner match [i] now becomes an outer match (on the periphery of the structure)! And we know a *single, non-shared, outer* match can be removed without creating orphans, if the other three matches it forms a square with are internal (each of the three matches is initially shared with two separate squares).

Between one and four matches destroy one square, depending on

- Where on the structure the matches are located
  - Outer (corner, edge) non-shared match
  - Inner match shared between adjacent/dependent squares
- What the support structure is (no orphans!)
  - Eliminate matches without destabilizing the structure

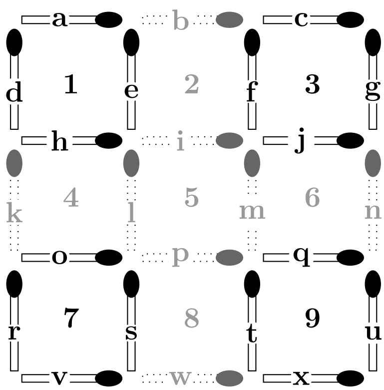
One match can belong to one square, or two squares (one match is ‘shared’ between exactly two squares).
One match cannot belong to three or more squares. One or two or three or four matches of one square can be shared.
All four matches of middle square [5] are shared: [i] with [2], [m] with [6], [p] with [8], [l] with [4].
Three matches of edge square [2] are shared: [f] with [3], [i] with [5], [e] with [1]. The only match that is not shared is [b], an outer (and by definition, unsharable) match!
Two matches of corner square [1] are shared: [e] with [2], [h] with [4]. Outer [ad] are not shared.

M: Previous solution figures will lead to these solutions!

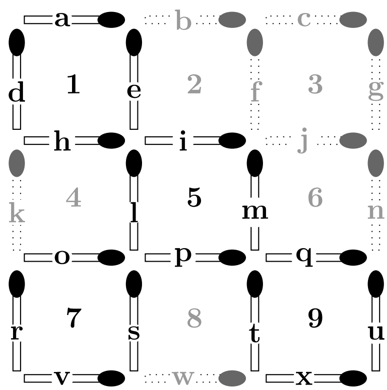
Legend: [Remove Matches] → [Break Squares].

To get **M1:**
G1, four matches & three squares removed. [pwmn] → [86].
G2, four matches & three squares removed. [klmn] → [46].
L1, two matches & two squares removed. [implkw] → [548].
L2, two matches & two squares removed. [klmnpw] → [468].

M2: **G3**, [bfjn] → [26]. **H2**, [bfk] → [24]. **L1**, [cgjfwk] → [384].



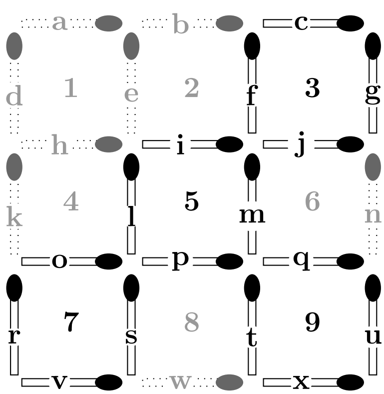
M1



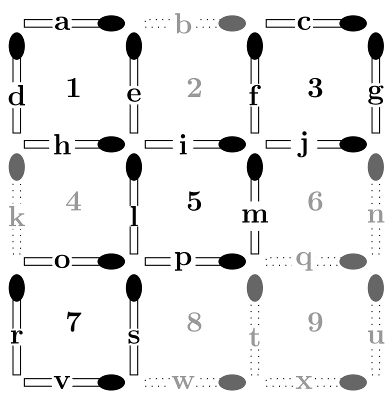
M2

M3: **L1**, [wdaehk] → [814].

M4: **L1**, [quxtwk] → [984].



M3



M4

You have *of course* realized that **M2**, **M3**, **M4** are the same solution, with suitable rotation!

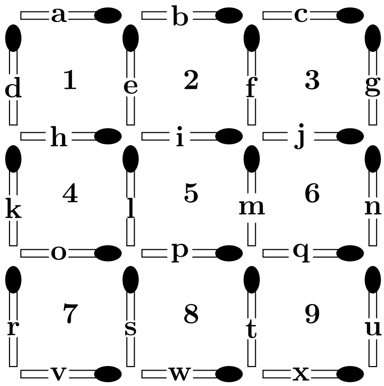
Complex, Square, Solutions

Without breaking, bending, overlapping matches, squares with different sizes *must be composites.*

Problems before now may have had composite shapes (squares or triangles). But a square was defined as being built from four matches (and a triangle from three). Problem statements also mentioned shapes/structures of the *same size.* Shape definition referred to (implied!) primitives, before this point in time.

While solving the problems in this section, you may wonder why a particular match is not orphaned. Remember to switch your thinking to composites! A match may be an orphan in context to primitive(s) it forms a part of; but may not really be an orphan because a composite it also forms a part of, gives it stability.

A: Identify the *total* number of simple and composite squares in the *original* (problem) figure:



Nine **small** (primitives): [1]–[9]
Four **medium** (four primitives each, medium squares overlap): {1245}, {2356}, {4578}, {5689}
One **large** (nine primitives): {123456789}, called {large}.
Totally, fourteen squares.
Note the {brackets} enclosing each *single* composite.
Small = primitive;
Medium, large = composites.

Primitive is the smallest shape that can be built using the fewest matches (four for a square, three for an equilateral triangle).

Simple composite (square or triangle) has no internal matches; only outer contour matches, to give it shape. It is a *single* shape, many matches per side. {abfmpokd} (medium simple composite) *does not need* internal matches [eilh]. Its (composite) area is four times a primitive area, but is a *single* square shape.

Normal composite (square or triangle) must have internal matches. There are 'shapes within shapes', definitely more than one shape.

The symmetrical figure has these unique matches:
[a]: Shared by [1], {1245}, {large}. Symmetrical: [cdgruvx]
[b]: Shared by [2], {1245}, {2356}, {large}. Symmetrical: [knw].
[e]: Shared by [1] and [2], {2356}. Symmetrical: [fhjoqst].
[i]: Shared by [2] and [5], {4578}, {5689}. Symmetrical: [lmp].

To understand how a group of matches is symmetrical, rotate and/or flip (horizontally, vertically) the figure. Symmetrical match *locations* will coincide.
Flip vertically: Location [a] will coincide with where location [c] was before the flip.
Rotate 90° anticlockwise: Location [d] coincides with where location [v] was before rotation. Then flip vertically: Location [d] coincides with where location [x] was before rotation and flip.

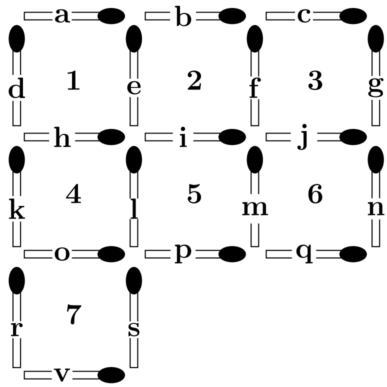
Page 38 gave match sharing rules for primitive structures:
Matches internal to the overall structure are shared by exactly two adjacent primitives (squares or triangles).
Matches that form the periphery of the overall structure are not shared (belong to a single primitive).

For composite structures, the picture is different:
Matches internal to the overall structure are shared by exactly two adjacent primitives (squares or triangles), and may be shared with other composite structures. In Problem **A**,
[h] is shared with three squares: [14], {4578}.
[i] is shared with four squares: [25], {4578}, {5689}.
Matches that form the periphery of the overall structure must belong to a single primitive, and may be shared with other composite structures. In Problem **A**,
[v] belongs to [7], is shared with: {4578}, {large}.
[w] belongs to [8], is shared with: {4578}, {5689}, {large}.

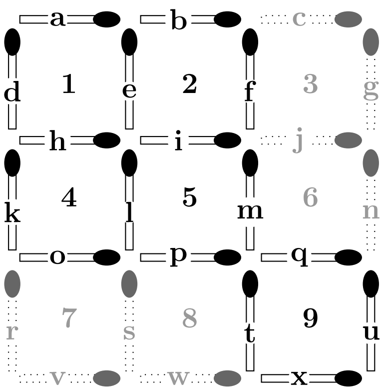
Removing a match belonging to multiple structures in a composite, destroys all these structures:
Removing [h] destroys [14], {4578}.
Removing [i] destroys [25], {4578}, {5689}.
Removing [v] destroys [7], {4578}, {large}.
Removing [w] destroys [8], {4578}, {5678}, {large}.

In any composite structure, periphery matches of primitives or composites

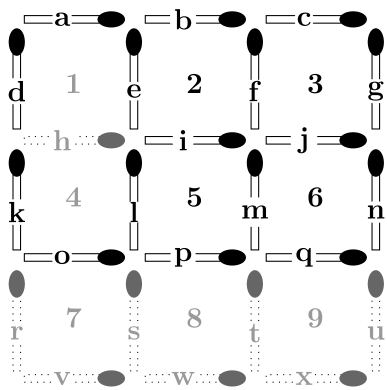
- Are definitely shared if they are not the periphery of the overall structure.
  [e] belongs to [12], {2356}.
  [o] belongs to [47], {1245}.
- May be shared if they form the periphery of the overall structure.
  [k] belongs to [4], {1245}.
  [q] belongs to [6], {2356}.
  [rsv] only belongs to [7].
  Matches [rsv] are not shared!



Note: All matches must form part of the periphery of some structure(s). Internal match [e] belongs to (and shares part of the periphery of) [1], [2], {2356}.



A1: [12459], {1245}



A2: [2356], {1245}, {2356}

A1: Remove [cg]: [3], {2356}, {large}, destroyed.
Remove [jn]: You cannot destroy the already destroyed {2356}, {large}. Only [6], {5689} destroyed.
Remove: [rv]: [7], {4578} destroyed ({large} destroyed earlier).
Remove [sw]: [8] destroyed (*not* {4578}, {5689}, {large}).

A2, **A3** remove the same seven matches ([rvswtxu]); destroy the same *six squares* ([789], {4578}, {5689}, {large}). Eighth match
[h] (for **A2**) destroys [1], [4], {4578}, *three squares.*
[i] (for **A3**) destroys [2], [5], {4578}, {5689}, *four squares.*

With simple addition,

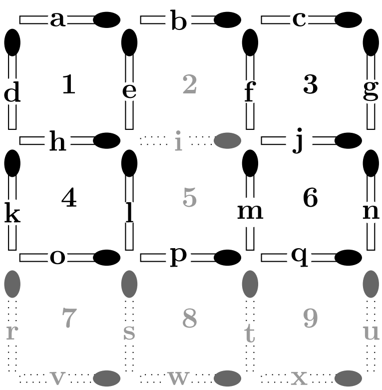
A2 destroys 6 + 3 = 9 squares; **A3** destroys 6 + 4 = 10 squares.

This is obviously wrong, as we can see six squares remain out of a total of fourteen (so only eight squares are really destroyed in either of solutions **A2** or **A3**).

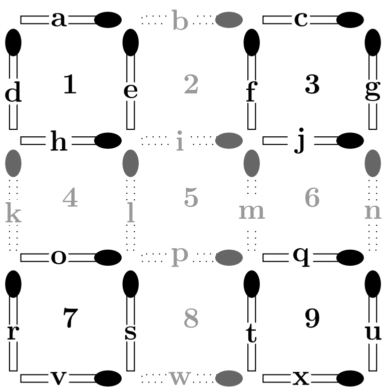
An expert like you can easily explain the anomaly:

In **A2**, [h] destroys [1], [4], {4578}, in *isolation*. If the other seven matches ([rvswtxu]) were removed first, any one of [rvwt] has already destroyed {4578} (since they are part of its contour).

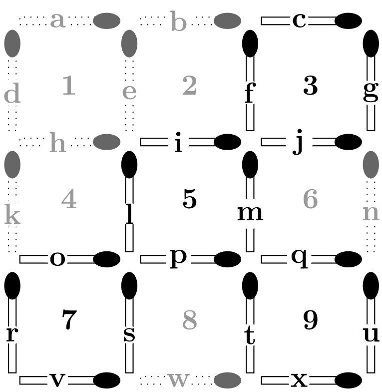
In **A3**, [i] only destroys [2], [5] ([w] has already destroyed {4578}, {5689}).



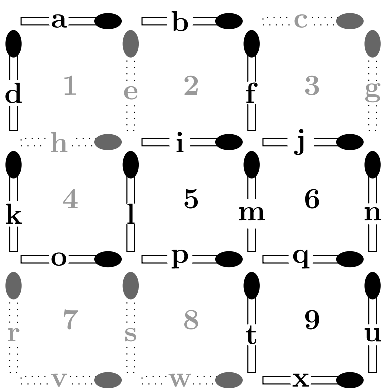
A3: [1436], {1245}, {2356}



B1: [1379]



B2: [3579]



B3: [569], {1245}

Composite solutions **B1**, **B2** are made only from primitives. They *are* primitive solutions **M1** (page 43) and **M3** (page 43)!

Some composite solutions may have equivalents in primitive solutions, already explored. We did not account for composites in those (primitive) solutions.

More analysis!
Instead of assuming a complete structure to be decomposed (remove matches), compose (*build*) the structure using matches.

In isolation, {large} needs twelve matches, {medium} needs eight matches, a primitive square needs four matches.

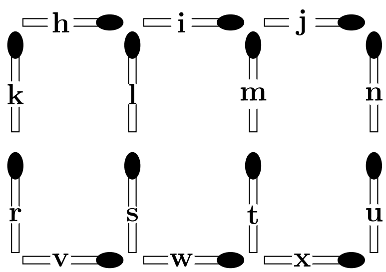
The problem figure has twenty-four matches. Solution figures created must be stripped down (skeleton) versions of the problem figure, and contain the required number of matches.

We only explore composite problems involving the removal of eight matches (*sixteen matches must remain in the solution*).

Consider a solution with {large}: Twelve matches have been placed, four more are available.
They must form two primitives at any two corners of {large} (and use two matches shared with the existing structure, per square)
or, form one primitive within {large} (no shared matches).

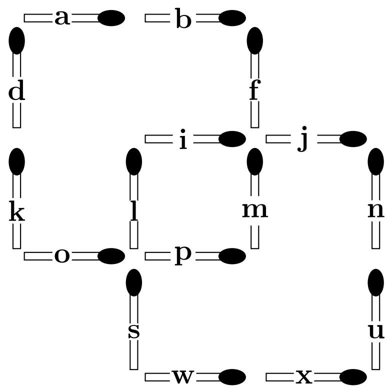
Consider a solution with {medium}, consisting of eight matches. Eight more matches are available.

Two overlapping {medium} squares share two matches (here, [himtwvrk], [ijnuxwsl] share [iw]) and an area equal to two primitive squares (here, [imtwsl]). This is a fourteen match structure.

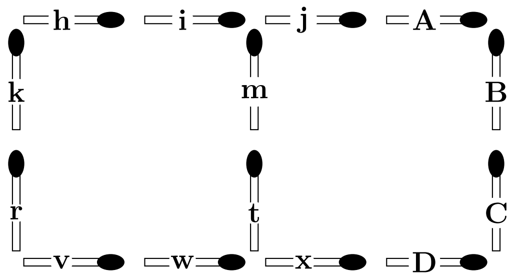


Two more matches must form a square within the structure.
Wherever they are placed within the {medium}, each additional match creates two primitives. Where these two matches are placed leads to solutions **A2** (page 46) or **A3** (page 47).

Two overlapping {medium} squares *do not share any matches*, but share an area equal to one primitive square (here, [impl]), a sixteen match structure.

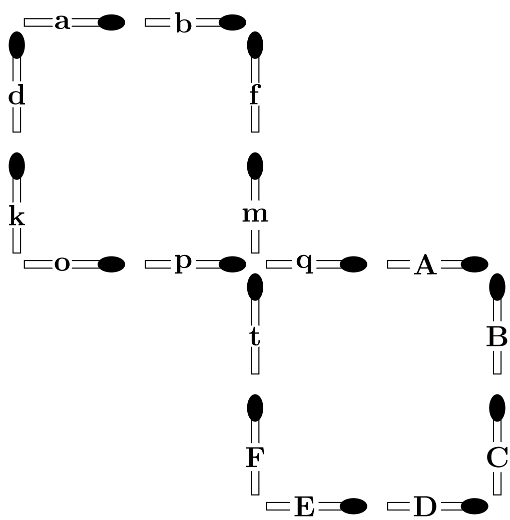


Non-overlapping [himtwvrk], [jABCDxtm] share [mt]. This is not a solution pathway, as matches [ABCD] stretch beyond the problem figure framework.



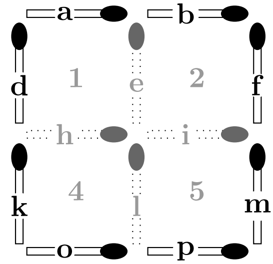
Do not lose focus! The goal needs a sixteen match solution *and* the solution figure must fit within the periphery of the problem figure! Forget this and you will be tempted to think of the fourteen match figure above as a solution (put the last two matches at any of eight corners of the two medium composites).

Non-overlapping {medium} squares touching only at one common corner (thus sharing no area or matches). Matches ABCDEF stretch beyond the problem figure framework.
This is not a solution pathway.

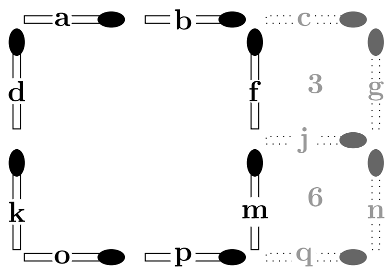


Consider a solution with one medium composite, {abfmpokd}. Eight more matches are available. Primitives can be placed,

Within: Two matches, [eh], create the first primitive, [1]. A third match, [i], creates the second primitive, [2]. A fourth match, [l] creates the third and fourth primitives, [45].



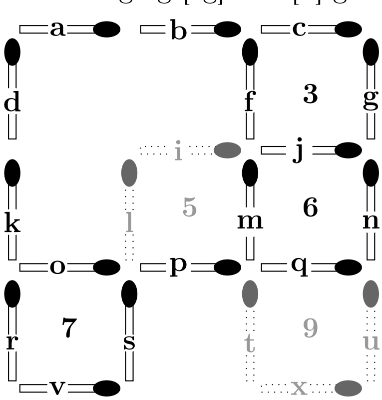
On the periphery: [cgj] create [3] (backed with {medium} shared match [f]). [nq] create an adjacent primitive, [6]; [j] is shared between primitives [3] and [6], [m] is shared between {medium} and [6].



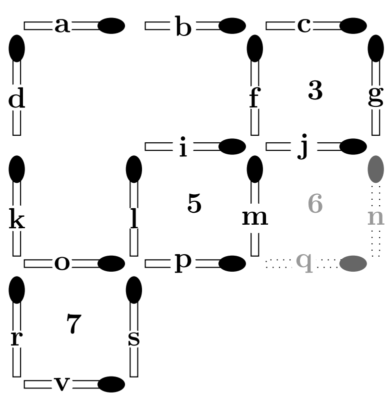
With four primitives within {medium} (twelve matches, five squares!), four more matches must form an isolated primitive (since a periphery square needs three matches; a periphery square adjacent to this one needs two more matches). This is Solution **A1**!

No primitives within {medium} gives new Solution **B4**. Interchanging [il] and [cg], [tux] and [rsv] gives **B3**.

One primitive within {medium} gives new Solution **B5**. Interchanging [cg] and [il] gives a solution similar to **B5**.



New Solution **B4**



New Solution **B5**

Orphans, Orphans, Everywhere!

This problem needs prior thought (analysis!) to solve efficiently. The aim (goal!) is not just utter destruction, but efficient utter destruction with the least amount of matches.

With no clue of what that least amount of matches is, only systematic analysis will prove your result is a (least amount of matches, utter destruction) solution.

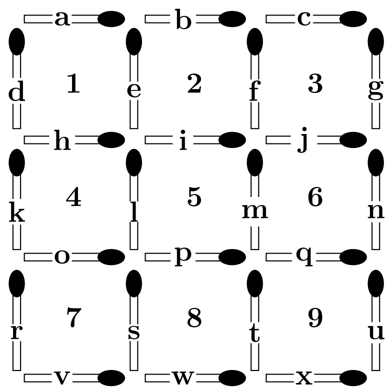
Some strategies to help achieve your goal:

- There is no advantage in destroying a lone structure if a shared match can destroy two.
- Destroy the most structures per match. For example, getting rid of a match belonging to (shared by) a primitive, medium, large, destroys three structures.
- Composite structures have more matches forming their contour/ perimeter/periphery, a primitive comprises the least matches. There is more opportunity to destroy larger structures, while destroying the smaller ones. Start by removing shared matches, to destroy two primitives. With luck, a shared (primitive) match will also be part of a larger (medium or large) structure.

C: Reference pages 39, 44.

A nine primitives problem figure. One (shared) match can destroy two primitives. So, a theoretical most efficient destruction can be achieved with five matches (four of them shared).

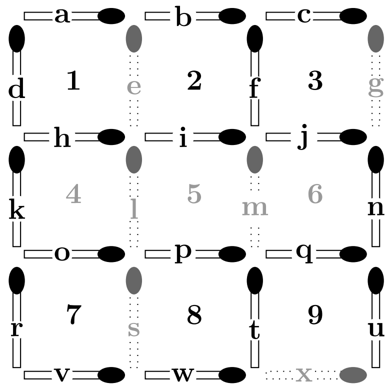
Test this out in practice.



Matches in the problem figure's center will do the most damage. [impl] each belong to two primitives and two {medium} squares.

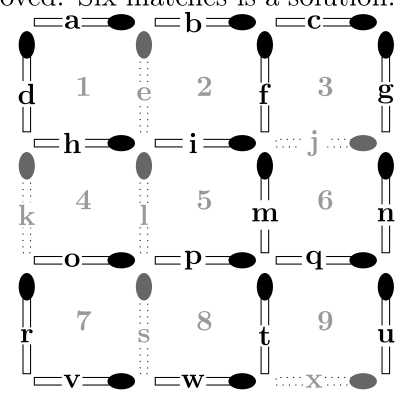
Result or Solution? You may get a result, but not necessarily a solution. A result adhering to all the problem's rules, qualifies as a solution. For example, say you need to win a game in exactly five moves (what the problem demands). If you win in less or more than five moves, you have a result, but not a solution!

Remove [l]: [1236789], {1245}, {4578}, {large}, remain.
Remove [m]: [123789], {large}, remain.
[123] are isolated from [789].
Remove [e]: [3789], {large}, remain.
Remove [g]: [789] remain.
Remove [s]: [9] remain.
Remove any one of [quxt].



Is this a six match result or solution? Here, success is defined by the least number of matches removed. Six matches is a solution!

Remove [e]: [3456789], {1245}, {4578}, {5689}, {large}, remain.
Remove [j]: [45789], {1245}, {4578}, {large}, remain.
Remove [l]: [789], {1245}, {4578}, {large}, remain.
Remove [s]: [9], {1245}, {4578}, {large}, remain.
Remove [k] and [u] or [x].



Too many solutions involving six matches: [ejpkru], [eltjor], [eltjkv], [ldfsqw], [ejpowx], [ejpkru], [eltjor], [eltjkv]. . .

Nothing new to learn, no new pattern to be discovered. We stop.

You are encouraged to try and find new patterns, new ideas.

Please send feedback! `info@problemsolvingpathway.com`

Enjoyed solving these problems? There are more in the books!

Found these puzzles difficult? Book I contains simpler ones.

Want more samples? Got a suggestion? Found an error?
Want to buy a 'Print On Demand' version of the books?
Send an email to `info@problemsolvingpathway.com`

Go to `www.problemsolvingpathway.com`, for more details.
Books I and II are 'print ready'. Scouting for a cost-effective printer with worldwide coverage. Suggestions are welcome.

To Get Lost Is To Find The Pathway

This book series teaches core problem solving concepts: Definition, exploration, sensing and verifying detail (via observation and analysis), separating fact from fiction, retaining relevant facts, thinking (logically, analytically, critically), stated versus implied information, investigating all possible solution paths while abandoning impossible ones, educated guesswork, understanding what is an acceptable solution, judging and deciding, implementing the optimal solution, learning from failure or inefficient solutions, seeking feedback, re-implementing a better solution.

Each self-contained book teaches a separate set of concepts. The problem solving apprentice is advised to sequentially follow the learning pathway, the expert can enhance specific problem solving concepts.

I: Puzzles & Patterns Puzzles, patterns, and mathematics teach analysis and investigation, the foundation concepts of problem solving.

II: Shapes & Contours Shape and matchstick puzzles explore all solution pathways, identify those that lead to solutions.

III: Games & Thinking Logical thinking is explored in the guise of parlor games. Lateral and creative thinking are introduced.

IV: Testing, Testing Go mad exploring these conundrums! Gain fresh insights and understanding of familiar tools and concepts.

V: Real World Tools Thinking, questioning, attention to relevant detail, intuition, judging & deciding, solving real world problems.

VI: Along The Pathway Defining the problem solving process, awareness of obstacles to problem solving, tips for the ace problem solver.

VII: Case Studies Analyzing the inner working of real world stories and problems gives insight to tackle any generic problem.