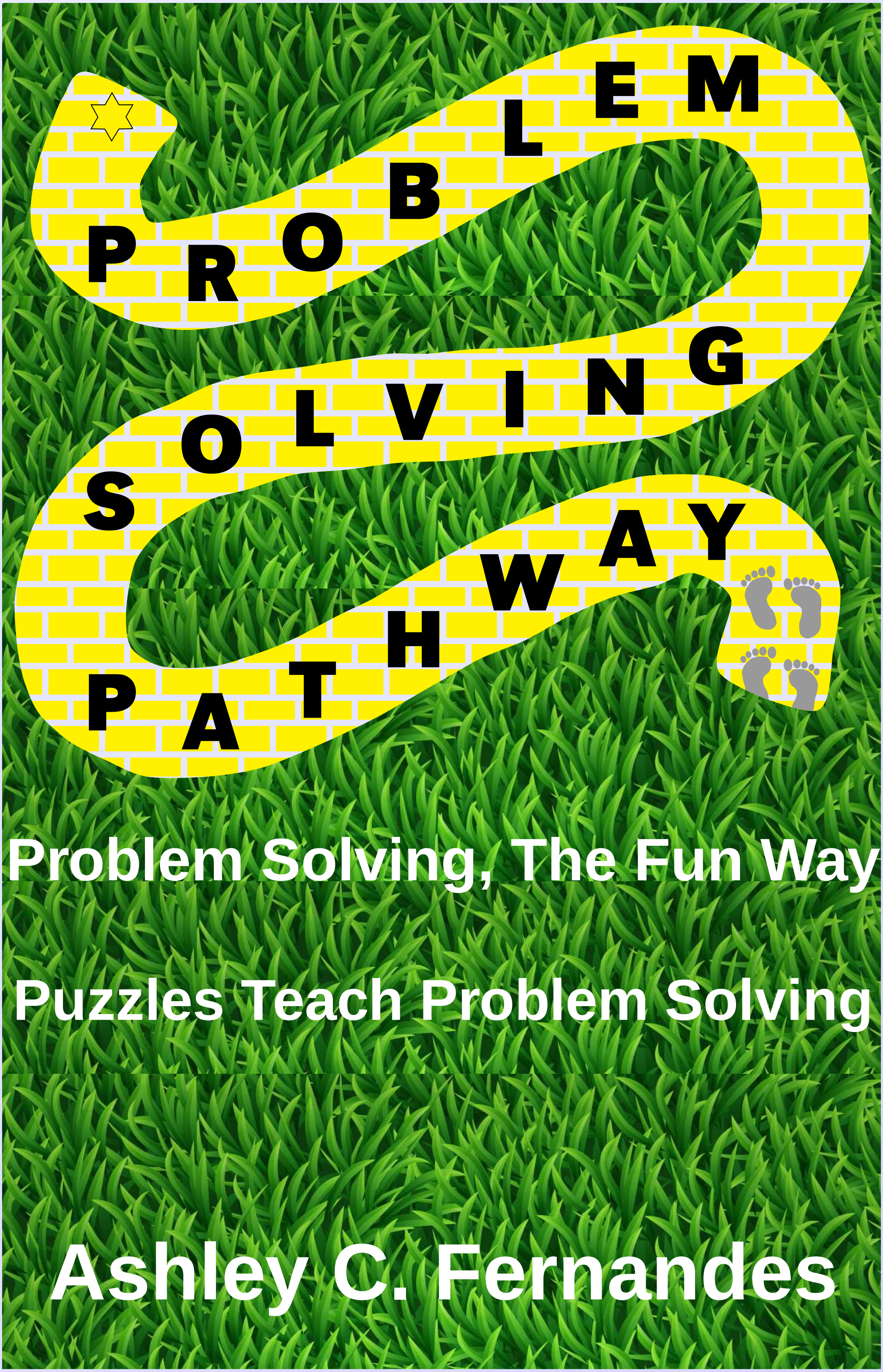

PROBLEM SOLVING PATHWAY

Problem Solving, The Fun Way

Puzzles Teach Problem Solving

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Problem Solving Pathway

Puzzles Teach Problem Solving

Problem Solving Pathway: Puzzles Teach Problem Solving

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Problem Solving Pathway

Problem Solving, More Than ‘Just’ Solving Puzzles

The easiest (and most fun!) way to learn basic problem solving is to treat puzzles as problems. These ‘problems’ are then solved in a *tightly controlled environment* (containing all relevant facts, rules and constraints needed to get an answer or answers).
Advanced problem solving investigates *loosely controlled* real world problems (with missing facts, and/or incomplete rules and constraints). These problems are difficult or impossible to solve without creating new relevant facts/rules/constraints that do not exist in the actual problem statement, or interpreting/enhancing the existing relevant facts/rules/constraints.
All kinds of problems need a clearly defined/understood goal.

We first understand the characteristics and differences between puzzles and problems. We next investigate how puzzles teach basic problem solving. Advanced problem solving is not covered.

The words ‘problem’ and ‘puzzle’ have many meanings, but we use the positive sense of these words. Problems and puzzles are:

- Obstacles or challenges (that need to be overcome by analysis, investigation, thinking) between problem and goal (solution).
- Situations to be tackled head-on, solved, maybe improved on; definitely not to be avoided, transferred to someone else to solve, or get frustrated over.
- Situations that *we* do not have solutions for, that are not well understood *by us*, that do not have known (*by us*) step-by-step instructions to solve them.

Investigate a problem, understand its inner working, ‘discover’ the step-by-step instructions needed to solve it, reveal all its solutions; convert it from ‘problem’ to ‘routine’.

By their nature, puzzles and problems challenge mentally, baffle, confuse. Solving them *tests and teaches* skills/tools like thinking, (logically, laterally, creatively, critically) analysis, investigation, educated guessing, questioning, making and testing hypotheses, judging, choosing, deciding… Trying to solve a conundrum, you *will* get stuck often, that is why they are called puzzles and problems! The true puzzle or problem solver must persist until they get unstuck, the joy is in finding solutions. Discovering answers/solutions builds confidence, patience, persistence.

Puzzle	**Problem**
A well-understood, usually *theoretical*, very tightly controlled situation. Having been worked out before, there are known ways to get an answer or answers.	Usually a *real world* unknown situation (no previous precedent has been set). There is uncertainty on how to proceed, what choices to make. Working out the problem completely will prove *if* solutions exist, and find them all.
Contains all relevant facts, constraints, rules (no extra information needed) to get at least one (and usually constrained to give exactly one) correct answer. Do not assume/enhance anything! A self-satisfied situation.	Filter out unwanted information. If relevant facts/constraints/rules cannot solve the problem or if ambiguous information exists, assume (*add* facts/constraints/ rules to the problem statement, and/or *enhance* existing ones). Zero or more solutions can exist!
Less to investigate, choose.	More to investigate, choose from.
Puzzles usually exist in a single domain (language, mathematics, logic, shapes, patterns. . .)	Real world problems usually exist in multiple domains. Sometimes it is difficult to know which domains the problem exists across!
The focus is on finding an answer by any means (even by guessing!), not on the understanding of how to get that answer.	While solutions are important, thinking and reasoning at every step and stage on the problem solving pathway is critical. A guess or luck needs eventual explanation.
Everyone working out the puzzle must always get the same answer(s). Assuming or interpreting is banned.	Different assumptions and/or interpretations of the problem or different (changing) circumstances may give different solutions.
Find answers by intuition, experimenting, random guess (understanding or justifying answers is not always possible); or by a series of logical steps (the answer and how it was found, clearly understood).	Find solutions by intuition, experimenting, educated guess, logic. Every solution and the steps used to obtain them must eventually be well understood and justified, to have truly solved the problem (eliminate randomness or intuition or experimenting).

Problem solving finds and explores all paths starting from the problem, and leading out. It understands what has happened on each path that has led to success (solutions!) or failure.

Chance and guesswork may find solutions without (by *our* definition) solving the problem! Only after understanding and explaining the mechanism behind the chance or the guesswork, can the problem be considered solved.

Solving a problem means *finding all existing solutions*. For a problem with no solution, solving it will prove that no solutions exist *here* (you solve the problem without finding any solution).

Just apply the formula!

Puzzles (problems, for the purpose of our study) can usually be solved using clever tricks, formulas, shortcuts, step-by-step instructions, that may not always be clearly understood.

The *problem solver* (you!) must think and investigate; must discover the 'how', the logic, clever tricks, formulas, shortcuts, step-by-step instructions, that solve a problem.

A *solution user* with no understanding of the problem or desire to learn problem solving, can just use the 'formula', follow a 'routine' series of well-defined steps (that you *discovered*!). They 'magically' get solutions without understanding how the formula or steps were arrived at or how they help solve the problem. They mechanically apply a bunch of memorized routines or steps, without thinking or understanding (or solving!) the problem.

Change in problem inputs (relevant facts/constraints/rules/goal) make the current solution invalid. Not knowing how to solve the modified problem (they never solved the original!), solution users must return to you (the problem solver) for a new magic formula.

You have already investigated the original problem, understood its inner working, identified all possible successes/solutions and failures. You now need little additional time and effort to solve the modified problem (with the new constraints)!

The problem solver *solves problems by thinking and investigating.* There is no 'road map' on how to proceed, what steps to take, what to do next. The thinking and investigation build the steps.

Problem solving (our definition): Explore *all* paths, find all solutions and results (avoid errors, if possible). Solution users implement defined steps *without thinking*, to get solutions.

Learning Problem Solving Versus Doing IQ Tests

The IQ test is designed to *test* thinking, investigation, the ability to choose among alternatives, specific domain knowledge, under stressful conditions (chiefly time constraints). It is not designed to teach these skills. An IQ test with answer choices (Multiple Choice Questions, MCQ's) can be aced without any thinking, only random guesswork and luck. Without answer choices, the test will only check if you know the correct answer.

Learning problem solving will *teach and test* thinking, analysis, investigation, judging, choosing, deciding, skills (among others). Investigate each problem in detail, to master problem solving. The emphasis is not to 'think fast and think correct', but to:

- Think slowly, and to understand your thoughts,
- Treat mistakes as learning experiences (avoid them from now),
- Find all possible paths starting from the problem; leading out,
- Identify all solutions; decide which is theoretically the best,
- Implement, test, improve the solution (practical conditions).

IQ tests bring quick satisfaction (for correct answers) or regret for (wrong answers). There is no incentive for careful thinking of multiple 'what if' scenarios'. Only the final answer is critical, not the method (path) to get to it. IQ tests assume you already have skills, they only test how well you have mastered these skills.
Solving problems effectively (exploring all paths leading to all possible solutions and results) takes effort and needs time. Each step on each path is a learning experience, and frequent failure is encouraged as a teaching tool. The emphasis is on learning something new each time, on applying old learning to a new problem and learning some more. You think more, you learn much more (than taking an IQ test).

IQ tests are sterile. If you know the principles (tools) behind similar questions, you only need apply routine formulas or logic. Problems are situations you have not encountered before. You may have encountered similar situations, you may know all the tools you will ever need. But you still must try various tools to find which will give solutions. On the way you will explore all paths, find results, perhaps make errors (and learn from them).

Learning problem solving teaches you how to solve puzzles and IQ tests. The reverse is not necessarily true.

Solve Puzzles To Learn Real World Problem Solving

In the quest to learn real world problem solving, you will start with an empty toolkit, gradually collecting tools/skills and learning how to use each tool (the techniques to use the tools). Novices must first understand each new tool/skill, learn how to use them (the techniques!) in trivial (puzzles/games) situations. Apprentices must take the same learning of using the tool/skill to more involved real world problems. This is the pathway to mastering problem solving!

Most classroom and book learning use topics like mathematics, statistics, probability, MENSA* conundrums, to teach problem solving. These topics/skills have a low 'like' factor.
Simple puzzles and games are fun activities for a novice to start learning problem solving. They are entertaining, engaging, thought-provoking—an ideal learning environment!
The aim is to gently introduce complex (real world) problem solving using baby steps (treating puzzles/games as problems).

The journey from novice to apprentice to master is slow progress. Key traits needed to gain mastery in problem solving are:

- Persistence: Invest a lot of effort and time.
- Resilience: Obstacles and frequent failure lead to rare success.
- Not letting a good solution stop your quest for a better one.
- Stepping outside your comfort zone. Skill mastery is not easy!

Allocating more resources (chiefly time and effort) to a problem increases the chances of solving it (finding all solutions, results).

Errors and results are not wasted effort! They are all learning experiences, even if they teach you where not to go and what not to do in the future.

Finding one solution is the puzzler's way, finding all solutions and results is what a problem solver does!

Solving puzzles is only the first step in the quest to learn problem solving! You must also understand how to solve different kinds (families) of real life problems, before you can claim to have truly have mastered the science of problem solving.

The **Problem Solving Pathway** book series teaches the novice using puzzles/games, and the mature reader using case studies.

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*MENSA is an organization for people with high intelligence.

Definitions & Clarifications

Most problem solving terms are used generically, or can mean different things in different pieces of literature. This discussion eliminates the ambiguity and confusion by rigidly defining each term that will be used to solve problems/puzzles in these books. If the terms baffle or the example does not clarify doubts, move on! Solving the actual problems/puzzles will aid understanding.

The terms **Problem** and **Puzzle** are synonymous. Problems *generally* have **Solutions**, puzzles (and parlor games) *generally* have **Answers**. Words 'answer' and 'solution' are synonymous.

Tool/Skill: A thing to use/do, to get from problem to solution.
Technique: How a tool is used. A hammer (tool) can be used to hammer in nails (Technique 1) or as a weapon (Technique 2)!

Stakeholder: Anyone who has an interest or stake (is involved and affected) in getting a solution to the problem. Types include:
Sponsor (usually upper management): Supports the search for solutions, allocates resources (money, manpower, equipment, time) to finding solutions to the (unknown) problem under study.
Problem owner (usually an individual): Is responsible for all stages of problem solving, including deciding which solution to implement among many (preferably after consulting the other stakeholders, or with domain experts). The person whose head is on the chopping block if anything goes wrong!
Problem solver (You!): Someone with problem solving expertise and experience, and with a toolkit full of tools and techniques.
Solution implementer: Tests each solution under real world conditions; verifies if each solution practical performs as it was theoretically expected to.
Solution user(customer): Uses the implemented/mature solution.

Constraints: Conditions a problem demands (legal) and forbids (illegal), of each valid solution.

Rules: Conditions of investigation, mathematics, calculation, logic; or steps of a process; or all legal moves in a game.

Goal: What problem rules and constraints define and demand. Only solutions and optimal solutions satisfy a problem goal.
A problem without a clear goal cannot be solved (assume facts, rules, constraints, but never the goal!).

Solution: An *absolutely correct* outcome of solving a problem (obeys all the problem rules and constraints, and *satisfies the goal*). For multiple solutions, decide which *one is optimal.*
You will not get a solution if you lack experience and/or expertise to find one. Learning problem solving will give you the expertise!
A problem with no solution is rare, and will not be studied here.

Optimal solution: The 'best solution to implement/use under current circumstances'. It could use the fewest resources (money, manpower, equipment, thinking effort. . .), be the fastest or most efficient or most practical, the easiest to understand/implement/modify, the one that has the widest stakeholder acceptance. . .
The optimal solution will (theoretically) best solve the problem (but must be proved to do so, after implementing and testing).
Puzzle problems must be solved fastest or with the fewest steps.

Error: An outcome that breaks one or more rules/constraints. Errors could lead to failure (no solution) or to an otherwise legal solution which is now wrong/unacceptable (due to the rule/constraint violation). Avoid making errors.

Result: Working out a problem until the end (no forward moves possible) and obeying all rules and constraints gives no solution because a *solution does not exist for this **path**.*

This explanation will be clear once you understand **paths**:
A problem with a result does not mean this problem has no solution! For a problem to have no solution, *all* paths from the origin point must lead to (termination point) results.
Ignore the possibility that an inexperienced problem solver could get a result on a path (that actually leads to a solution).

Tightly constrained problem: *usually controlled environment* of puzzles and games. All relevant facts, rules, and constraints essential/needed to solve the problem are provided (stated or implied). The goal is clearly defined. You *are not allowed* to make any assumptions.
Everyone reading the problem must have a single understanding/interpretation of all the relevant facts/rules/constraints (what is permitted, what is illegal) and solution expectations (the goal).
Loosely constrained problems (usually real world) may be difficult or impossible to solve without assuming/interpreting/enhancing/adding, available relevant facts/rules/constraints).

All (tightly and loosely constrained) problems must have a clear goal. A problem with a vague or non-existent goal cannot be investigated (problem cannot start!), therefore has no solution.

All problem **stages** and **steps** need analysis, decision making!
Investigation is analysis. The word 'analysis' is deliberately avoided for the pathway stage, as actual analysis must take place within every problem solving stage and step.
Investigation and analysis (and any problem solving **stage** or **step**) needs thinking skills!

Choosing is deciding (and vice versa). When there is doubt or uncertainty on how to proceed or which path to follow (at almost every problem solving stage and step!), you must choose among alternative steps or paths.

Decision making is only needed where multiple acceptable solutions (correct answers!) exist. You must decide (a choice!) which single solution to implement (the optimal or 'best under current circumstances' solution). Evaluate (judge!) the merits and demerits (a comparison) of each solution. The outcome of the judgment is the decision.

Choosing and deciding demands multiple options (alternative steps, paths of travel, solutions); you must choose/select one.
A unique option is a 'no choice' option, since it must be selected.

The **problem owner** decides! With maximum stake in the optimal solution, this stakeholder has the final say.
The problem owner lacking technical expertise must rely on other stakeholders or maybe an *external domain expert** for advice.

After sufficient analysis, decide! Good analysis eliminates bad decisions, non-optimal decisions can be corrected with hindsight (and usually with little additional time/effort/resource penalty).

Domain expert*: Someone with experience and expertise at solving this type/family of problems. Domain experts are:
Internal: One of the types of stakeholders.
External: Not a stakeholder.

An internal domain expert has a stake in identifying the 'best under current circumstances' (optimal) solution.
An external expert does not have a stake in the solution itself, but has a reputation (of being a domain expert!) to uphold.

Problem Solving Pathway is a framework of:

- *What to do* (stages to follow) to solve any problem,
- *How to do it* (tools/skills to use, steps and paths to follow).

Use the framework to solve most problems effectively (identify *all possible paths*, find *all possible solutions*). Keep adding to the framework, to solve more types/families of problems.
Problem Solving Pathway is *different from* ***path*** (see below).

Stages of the problem solving pathway: Defining the problem and goal, Investigating, Finding all solutions, Judging/Deciding, Implementing, Monitoring, Re-implementing.
Define, **Investigate**, **Judge**, **Decide**, **Implement**, **Monitor/ Evaluate/Feedback** (stages) are also tools!

Steps: A gradual progress (the detailed procedure) to solve a problem. All problem **stages** have many incremental steps.

Path: All problems must start at a single 'origin/entry' point (the problem statement), can end in multiple 'termination/exit' points. Each *complete route* of **steps** from origin-to-termination, is a *separate* path. Three types of terminations define three kinds of paths:

- **Solution** (*the only path leading to success*!),
- **Error** (rule/constraint violation, leads to *failure*),
- **Result** (*solution does not exist for this path*).

Solutions, errors, or results, are **outcomes** of solving a problem (assuming you do not abandon the problem midway).

Paths can branch out anywhere from the origin to just before termination. Branches can branch further, can rejoin the main path they broke away from (this would be a detour to the main path), can join other branches or paths. In the entanglement of paths criss-crossing or branches diverging and merging, consider each unique end-to-end path, and its steps.
Exploring all end-to-end paths must find *all* successes (solutions, that satisfy the goal) and failures (results, errors) that exist for this problem. If all end-to-end paths terminate in results, the problem has no solution. Assuming no error, each solution or result has a unique path from problem start (origin point) to successful outcome (termination point).

An Example Explaining The Definitions

"Win a chess game in six moves" is a loosely constrained problem statement because an efficient win in less than six moves is an undefined outcome (Is it a valid solution?). This is a typical real world problem, where you need to assume something (choose if a 'less than six moves win' is a solution).
A tightly constrained problem statement: "Win a chess game in exactly six moves." This problem statement has two constraints,
C1: You must 'win', C2: In 'exactly six moves'.
Implied: You must obey all the rules governing the game of chess itself. Break one or more chess rules (an illegal movement of a piece, or cheating) or constraints, and you have an error.

This problem statement is for illustration only. Ignore the fact that six moves is too short for a chess game to have an outcome.

Games completing with no rule violations; in exactly six moves and a win (so no constraint violations!), are solutions.
Games completing in a draw/defeat and no rule violations, can be *considered* results. Actually they are not, as they violate C1.

Games completing in more or less than six moves and a win, are errors (violation of C2).
Games completing in exactly six moves and a draw/defeat, are errors (violation of C1).
Games completing in more or less than six moves and a draw/ defeat, are errors (violation of C1, C2).
Games breaking rules and completing in exactly six moves and a win, are errors (violation of rules).
Games breaking rules and completing in more or less than six moves and a win, are errors (violation of rules, C2).
Games breaking rules and completing in exactly six moves and a draw/defeat, are errors (violation of rules, C1).
Games breaking rules and completing in more or less than six moves and a draw/defeat, are errors (violation of rules, C1, C2).

Any unique set of six steps/moves starting from the problem origin (new board, where the game has not yet started) to the termination (win or loss or draw) is a unique path.
An incomplete game (*abandoned* midway before the game is over), has no outcome (win or loss or draw), and is a situation that is unacceptable to the true problem solver!

An optimal solution to this chess game could be the least amount (or the least valuable) of your pieces captured, the most amount (or the most valuable) of your opponents pieces captured...
If the optimal solution needs retaining the most amount of your pieces, multiple paths could lead the same one optimal solution (retaining same amount and types of pieces), or different optimal solutions (retaining same amount but different types of pieces).

In this example, a 'less than six moves' win is a C2 error, and is more efficient than an 'exactly six moves' solution. Priority is achieving the goal, efficiency and all else are less important. You must obey all rules/constraints to get a (*here* inefficient) solution.

Most chess games allow a fixed *time constraint* to make a move.

Progress In Stages

The problem solving pathway is broken up into seven stages. Each stage can use multiple tools, and the tools themselves could be used in steps within any stage.

1. **Define the problem and the goal of the solution.**
   - Understand the problem statement unambiguously.
   - Only keep relevant facts, rules, constraints (for the goal).
2. **Investigate the problem.**
   - Explore all paths from problem origin to termination.
   - Use multiple tools/skills/ideas/clues.
3. **Find all possible (multiple) valid solutions.**
   - Follow all paths out from the problem.
   - Find all solutions; other paths are learning experiences.
4. **Judge and decide.**
   - With multiple solutions, judge the pros & cons of each.
   - Decide (a choice, a risk) the optimal solution.
   - Improve bad/inefficient decisions while re-implementing.
5. **Implement, roll up your sleeves and do it!**
   - Is the theoretical optimal solution also so in practice?
   - Identify faulty judgment, errors, solution improvements.
6. **Monitor to evaluate, feedback to improve.**
   - Hindsight, a wonderful teacher for solution improvement.
   - Correct deficiencies, fix flaws, improve existing solutions.
7. **Re-implement for continuous solution improvement.**
   - Experience and better facts, give better solutions.
   - Again monitor, evaluate to further improve the solution.

Tools Down The Pathway

Prominent tools listed here cannot solve every kind of problem. Find new tools, use existing tools to solve new kinds of problems. One tool can have many techniques (ways to use the tool).

- **Relevent facts.** From the problem statement. Know where to find more (authentic reference material, domain experts).
- **Ask relevant questions.** Find more relevant facts.
- **Experiment, make educated guesses.** Only if relevant facts are missing. No random guess. Prove your guess is fact.
- **Collective wisdom tools**: Brainstorming, crowdsourcing.
- **Special domain knowledge.** Medicine, science, finance, law, mathematics (numbers/calculation/shapes), language...
- **Patterns and connections between relevant facts.**
- **Thinking.** Logically, Laterally, Creatively, Critically.
- **Memory.** Impression and recall. Combining different (but similar) experiences, and assuming they are a single incident. False memories (unconscious lying).
- **Detail-oriented.** Sensing what is hidden, implied (beyond the obvious). Awareness of tiny (often overlooked) relevant facts that can improve investigation, give a better solution.
- **Time management.** An investment during the learning phase. Less time needed to solve real world problems.
- **Precise Language & Communication.** Avoid ambiguity, misinterpretation, loss of understanding.
- **Divide & Conquer.** Break a big problem into smaller tasks.
- **Walk-through.** Try all choices, paths; eliminate impossible ones, prioritize promising ones. Find all possible solutions.
- **Walk back!** For problems with many paths but a unique solution. Start at the solution, and trace back the logical steps toward the problem. Probably you will eliminate result paths and make less errors, be more efficient than trying all paths leading out from the problem.
- **Prior experience/expertise may help solve a problem.**
- **Change of perspective.** Look at a problem from the points of view of different stakeholders.
- **Drawing figures, diagrams, models, doodles.** Fishbone, Venn, Decision Tree, Mind Map, charts, graphs, tables...

Problem Versus Puzzle, Explained

This puzzle deals with simple logic, combinations, sequences. Look for an obvious, easy and non-tricky answer.

This conundrum is first explained as a loosely constrained puzzle statement, then as a tightly constrained problem statement. The puzzle answer and the problem solution are explained next.

The puzzle (like all puzzles) will seek to find an answer *somehow*. The problem will seek to understand exactly how the solution is arrived at. All tacit (implied) facts will be brought to the level of conscious (obvious) thought, all paths will be explored, multiple solutions (if they exist) will be found.

Loosely constrained puzzle statement

A wolf, a sheep, and a bundle of grass need to be transported across a river, by a man in a boat. Unsupervised, the wolf will eat the sheep or the sheep will eat the grass. The boatman can ferry only one item at a time, beside himself. How does he succeed?

Tightly constrained problem statement

While crossing the water no swimming, floating with the current, ropes or props, or 'shortcuts' are allowed. The man and anything else being ferried must always remain inside the boat at all times.

A wolf, a sheep, and a bundle of grass need to be transported across a swift flowing river, using a boat and nothing else. The boatman can ferry only one item at a time (beside himself). Only he can row, so he must be present in the boat on all trips.

Unsupervised, the wolf will eat the sheep, the sheep will eat the grass. All three are safe if the man is present to supervise them.

Tying the goat so it cannot reach the grass or wolf so it cannot reach the goat, is not allowed (there is no rope, remember?).

All three must be transported across the river intact (giving little grass to the sheep while ferrying the wolf is not allowed). No creature can swim, the grass will not float on water.

Assuming the task can be accomplished and he can make as many trips as needed, how does the boatman succeed?

Carnivores and Herbivores

This *puzzle* is easily solved for **W**olf, **S**heep, **G**rass, **M**an:
W&**S** *or* **S**&**G**, cannot stay together unsupervised (**W**&**G** can).
So (obviously) **S** must make the first trip.
For the next trip: Take **W** (**W** will eat **S** on the other bank), or take **G** (now **S** will eat **G** on the other bank). How to proceed?

Since the problem states a solution exists, it will quickly be clear that you must bring something back (on the first return trip).

The entire puzzle is solved by common sense:
Take **S** to the other bank, return alone.
Take **W** to the other bank, return with **S**.
Take **G** to the other bank, return alone.
Take **S** to the other bank, *Solved*!
This answer (or a slight variation, if you got the other answer) is how you solve puzzles. Since this puzzle can be solved in only one way, the unique answer must eventually become obvious.

Only after you realize that the boat can carry the man and one thing else on its return trip, will you get an answer to this puzzle!

All this is puzzle solving, not problem solving! Unless you *know* what you did (make implied understanding conscious thought), you will struggle with the next similar problem.
An involved problem differs from a puzzle in that you completely understand all the implications of facts (stated and implied), constraints, rules; and you identify the real goal.
Here is one implied (not stated in the problem) fact: The puzzle never said **W** will not eat **G**. Only accept facts (stated, implied) in *this* problem statement, not your own general knowledge (carnivores do not eat grass)! But you will quickly realize that progress is only possible assuming (*implied*) **W** cannot eat **G**!

Understand the true meaning of this obvious fact, and its implied meaning: Boat carrying capacity is two, **M** and one more thing. Capacity implies ‘maximum’ (or lesser) limit the boat can carry. The man need not ferry one thing every time. *Implied*: Full boat capacity ($\mathbf{M} + 1$) is desirable on a forward trip, to finish transport using the least amount of trips. Minimum boat capacity (only **M**) is desirable on the return trip, for the same reason.

Understand the true goal: Not transport, but *intact* transport.

A scientific (*problem solving*!) approach to this puzzle follows!

Legend for solving this conundrum:
Boat Trips: **1F** (1st forward trip); **2R** (2nd return trip)…
Wolf (**W**), Sheep (**S**), Grass (**G**), Boatman (**M**).
Locations: Near Bank (**NB**), Boat (**B**), Far Bank (**FB**).
Directions of travel: **NB**–to–**FB** (→), **FB**–to–**NB** (←).

Goal: Transport with zero damage. Boat constraint: Man + one.

The solution obviously needs multiple trips, but how many, and who goes first? And what are the fewest amount of trips?

At each step, check every combination/choice/path (no forgotten/overlooked/neglected path). Paths can lead to success or failure.

Eliminating obvious impossible combinations in the beginning leaves a smaller problem to solve! With only three choices here, who will make the first trip, going forward (**1F**)?

- Path 1: **W**? Impossible, **S** will eat **G** on **NB**.
- Path 2: **G**? Impossible, **W** will eat **S** on **NB**.
- Path 3: The last choice (**S**) *must* be true, since the problem statement specifies that the task can be accomplished (at least one solution must exist). So **W** cannot eat **G**, an *implied fact*! Given the problem constraints, **1F** *must logically* involve **S**.

Who takes the next trips hinges on a few more implied facts:

- Boat capacity: **M** + 0 (minimum) *or* **M** + 1 (maximum).
- The boat need not be empty on its way back (return trip)!
- **M** can make many trips, but fewer are optimal and desirable.
- **W** & **G** can remain unsupervised anywhere.
- Other combinations (**S** & **G**, **W** & **S**) need supervision.

Look beyond the obvious. In problem solving, some facts are stated, others are implied. The solution table is self-explanatory.

	Path 3a			Path 3b		
Travel Direction	**NB**	**Boat**	**FB**	**NB**	**Boat**	**FB**
Problem	**GWSM**	—	—	**GWSM**	—	—
1F →	**GW**	**MS**	—	**GW**	**MS**	—
1R ←	**GW**	**M**	**S**	**GW**	**M**	**S**
2F →	**W**	**MG**	**S**	**G**	**MW**	**S**
2R ←	**W**	**MS**	**G**	**G**	**MS**	**W**
3F →	**S**	**MW**	**G**	**S**	**MG**	**W**
3R ←	**S**	**M**	**WG**	**S**	**M**	**WG**
4F →	—	**MS**	**WG**	—	**MS**	**WG**

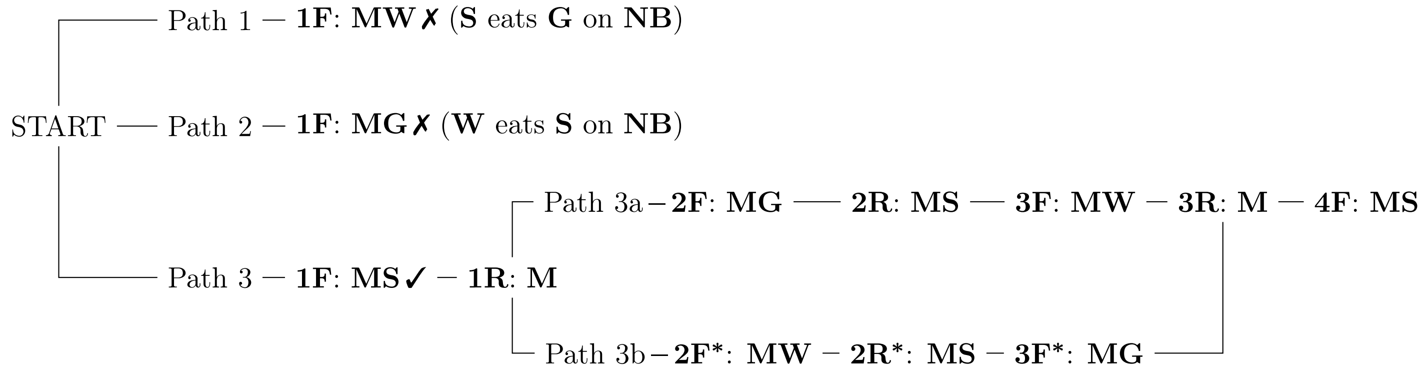
To solve this problem, you must ‘find’ two implied facts: **W** will not eat **G**; **B** can ferry something on return trips.

Path 1 and Path 2 lead to results.
Path 3 branches into two unique (end-to-end) paths. The branch (a detour!) merges back into the main path after three unique trips (steps): **2F*** ferries **W**, **2R*** returns **S**, **3F*** ferries **G**. Both paths have the same amount of trips, merge into a *unique solution*. Boat occupants differ only on three trips: **2F-2F***, **2R-2R***, **3F-3F***.

Main end-to-end path 3a: **1F-1R-2F-2R-3F-3R-4F**. Branch (detour) path 3b: **1F-1R-2F*-2R*-3F*-3R-4F**.

This diagram (called a decision tree) shows how choices connect and/or lead to the next choice in a chain (path!). The legend shows who is in the boat at all times; it could also include who is on either bank.

It is not usually possible to classify problems so neatly, and to solve them so linearly.



Investigation never stops!
Stated relevant facts: Who needs to be transported, the boat capacity, the goal, the fact that unlimited trips can be made (the problem never asks for the least amount of trips). All three cannot swim or float in the water. Any combination of one or more is safe if the man is present. Only the man can row, so must always remain in the boat.
The swift flowing river is a fact, but is *irrelevant* to this problem since swimming or floating is forbidden.
Implied facts: If the boat is on either bank, whoever or whatever is on that bank is safe (the man is present). If the man is rowing (in the middle of the river), only **W** & **G** can remain together on either bank. Any one of the three items can remain alone on any bank. At all times, three locations must be stable: **NB**, **FB**, **B**. All three locations must always have **M** where **W** & **S** or **S** & **G** are present (and this is clearly seen in the solution table).

There is no limit to the amount of investigation you can do for this problem. Let your imagination loose, and try to poke holes in the rules or constraints, try to find new ways around the problem, try 'what if' scenarios...
Can the animals be in the water with their forepaws resting on the boat rim, and paddling with their hind legs? Technically they are not in the boat, but just being dragged along. So their weight is not added to the boat carrying capacity. *Not allowed*, the general problem statement specifies that everything must remain in the boat at all times.
Just put two things in the boat with the man, and somehow manage! *Illegal*, you must respect the boat constraint.
A brilliant suggestion: Let the sheep rest on the wolf's shoulders, in the boat. Technically the sheep is not in the boat (only the wolf and man have their limbs *in* the boat). Common sense will tell you they are both in the boat, and in any case both their weights will add to the boat carrying capacity. *Illegal.*
Thinking up any ridiculous or absurd scenario will violate some constraint of the problem. That is why this problem can easily be solved even by small children. They try everything imaginable and then stumble on the only thing that is allowed, and that leads them to the unique solution.

Now that you have investigated the problem so thoroughly, any changed conditions (facts/constraints/goal) can quickly give a new solution. For example all other things being same, find all solutions with the boat capacity of three.
You know there are finite possible combinations of things to ferry, that some things cannot remain together without supervision. List all combinations at every step, reject illegal combinations, test all legal ones and retain combinations that lead to solutions.

NB/Boat/FB (at start, WSG/M/·). Five scenarios are possible:

Trip	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
1F	W/MSG/·	W/MSG/·	S/MWG/·	G/MSW/·	G/MSW/·
1R	W/MS/G	W/MG/S	S/M/WG	G/MW/S	G/MS/W
2F	·/MSW/G	·/MGW/S	·/MS/WG	·/GMW/S	·/MSG/W

Although not asked, let us define efficiency for this problem. All five scenarios have the same amount of trips (Amount is not a criteria). Weight seems the only criteria/measure to define efficiency *in this problem.* Only in Solution 3 will **M** row back alone on **1R** and have only one companion for **2F**.

M is always in the boat!

- Solution 1: **S** remains in the boat for all trips.
- Solution 2: **G** remains in the boat for all trips.
- Solution 3: The most efficient trip.
  Trip **2R**: **M** can return with **W** or **G** (as **B** capacity of three for **2F** will not be exceeded). Without passengers for **2R**, **M** ferries less load/weight, (the most efficient trip!).
- Solution 4: **W** remains in the boat for all trips.
- Solution 5: **S** remains in the boat for all trips.

What if the boat capacity was four? One trip and transport is complete! No investigation needed, so this is *not a problem* by our definition.

Detailed investigation has morphed our one page answer to a puzzle, into a four page solution for a problem!

Based on this investigation, think of other combinations with different types of items to be ferried, letting more than one person row the boat, changing boat capacity, defining subsets of items that can live in harmony with each other but not with others…

Follow The Yellow Brick Road

Following every stage of the Problem Solving Pathway combined with careful thinking and analysis at every step, will guarantee you find all solutions and identify the optimal solution.

This text is a consolidated view of the problem solving pathway stages and the tools used. It is extracted from the ‘Problem Solving Pathway’ series of books, which explores the ‘fun’ aspect of problem solving.

The first four books teach aspects of problem solving, using fun activities like puzzles and games. Here one finds constrained ways of solving a problem, therefore defined kinds of solutions that are acceptable.
The last three books apply insight gained by solving puzzles and playing games, to real world problems. Here, loosely constrained problems may have many ways to solve them. Acceptance of a solution may not only depend on how optimal or practical it is, but also on what real world humans will accept and implement.

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To Get Lost Is To Find The Pathway

This book series teaches core problem solving concepts: Definition, exploration, sensing and verifying detail (via observation and analysis), separating fact from fiction, retaining relevant facts, thinking (logically, analytically, critically), stated versus implied information, investigating all possible solution paths while abandoning impossible ones, educated guesswork, understanding what is an acceptable solution, judging and deciding, implementing the optimal solution, learning from failure or inefficient solutions, seeking feedback, re-implementing a better solution.

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